Rational Inattention Choices in Firms and Households *

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Abstract

Standard policy analysis often assumes that macroeconomic conditions and policy news have common implications for households and firms. Yet surveys show that households associate higher inflation with lower output, while firms associate it with higher output. This paper develops a rational inattention model and shows that households optimally attend to real income, while firms attend to nominal marginal costs. Heterogeneous attention choices generate the opposing survey views and divergent responses to communication policies. Moreover, their attention is endogenous and interdependent, leading to state-dependent transmission of shocks.

Keywords: Rational inattention; Expectation formation; Firms; Households; Central bank communication.

JEL classification: D83, D84, E31, E32, E58.

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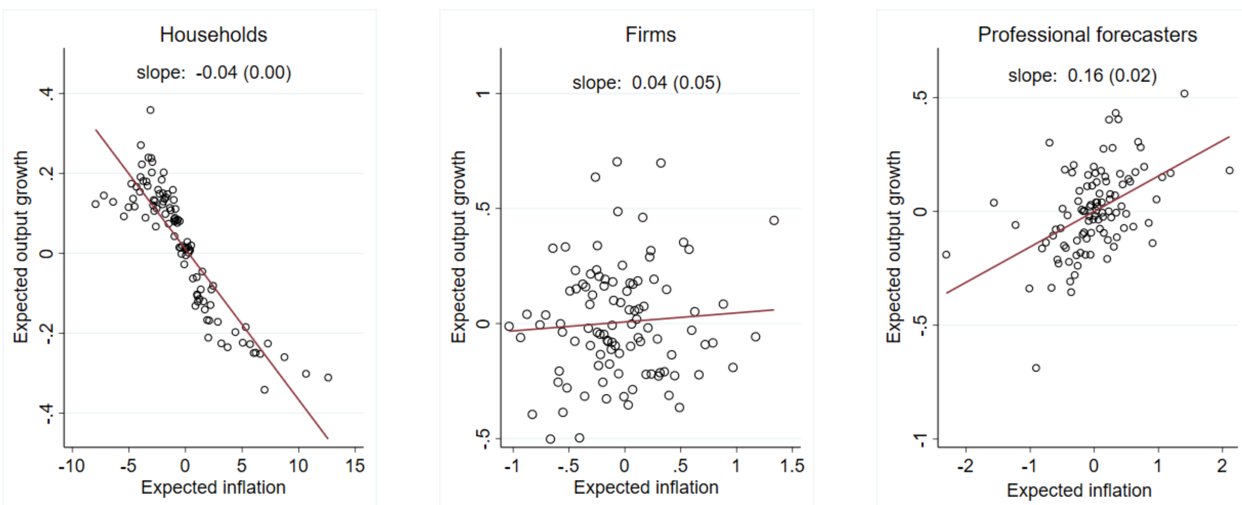
1 Introduction

The transmission of policy depends on how decision makers in the economy understand macroeconomic conditions and policy news. Standard policy analysis often assumes that households and firms draw similar inferences from the same macroeconomic data and policy news. For example, they are expected to agree on what higher inflation means for the economy, and to draw similar conclusions from policy interventions. This abstraction is far from innocuous. Survey evidence reveals a systematic divide in how households and firms understand the economy.

This divide is visible in Figure 1, based on Candia et al. (2020), which plots joint expectations over inflation and output growth for different economic agents in the United States. Households associate higher expected inflation with lower expected output growth, i.e., a supply-side view.[1] In contrast, firms and professional forecasters associate higher expected inflation with higher expected growth, i.e., a demand-side view.[2,3] These contrasting views matter for policy because the same economic condition or policy news may lead different agents to draw different inferences.

Figure 1: Correlation between expected inflation and expected output



Notes: Each panel plots the cross-section of forecasts of output growth and inflation after removing time-fixed effects. I present the resulting correlations in binscatter form for each agent. Data sources: Michigan Survey of Consumers, The Livingston Survey, The Survey of Professional Forecasters.

This paper takes these contrasting views seriously and asks two questions: what drives the different views by households and firms, and what do these differences imply for macroeconomic dynamics and policy transmission? Answering these questions requires moving beyond the full-information benchmark, but imperfect information should not be imposed arbitrarily. Ex-

[1]The negative association by households is labeled as a supply-side view, as supply shocks are expansionary for output and reduce inflation, leading to the negative comovement between output and inflation.

[2]The positive association is labeled as a demand-side view as demand shocks are expansionary for output and inflation (Candia et al., 2020).

[3]The cross-sectional patterns are consistently observed across various countries (Candia et al., 2020) and in randomized controlled trials (Coibion et al., 2021, 2023). Moreover, all these patterns also hold when controlling for individual-level fixed effects (see Appendix A).

ogenously assigning different information sets can help account for heterogeneous views, but it leaves open why these information differences arise in the first place. I therefore model information as endogenous: households and firms are rationally inattentive à la Sims (2003) and acquire the information most valuable for their own decisions. I show that the optimal attention choices of households and firms are *heterogeneous*, *endogenous*, and *strategic*. These heterogeneous optimal attention choices generate different partial information sets, leading to the observed heterogeneity in views, and divergent responses to communication policies. Endogenous and strategic attention lead to state-dependent shock transmission, I examine the implications for business-cycle fluctuations and monetary policy.

Preview. The full model shares the same core micro-foundations as the textbook New Keynesian economy with supply and demand shocks, except for one primary modification. I assume both households and firms are rationally inattentive, in that they can choose *"what"* and *"how much"* information to acquire, subject to an attention cost which increases with the informativeness of the signal.[4] Moreover, for ease of discussion, I do not assume exogenous nominal rigidities, rather price rigidity arises endogenously from firms' information frictions.[5]

The analysis proceeds in two steps. I first characterize the mechanism analytically in a static model, a special case of the full model without its dynamics,[6] where the results are closed-form. I then solve the full dynamic model numerically.

I first focus on the *"what"*. When attention is costly, optimizing agents allocate it to the information most relevant for their decision problems. Households make consumption-saving decisions, so information on the path of real income would be particularly useful: information about the current real income affects the consumption-leisure trade-off, while information about current versus future real income governs the strength of intertemporal substitution. Households therefore choose to learn about their real income. Firms, on the other hand, choose to learn about their nominal marginal cost, as their optimal price is a markup over the nominal marginal cost. In this model, with labor being the only input, nominal marginal cost equals the nominal wage.

As households and firms learn about different endogenous variables, they are differently informed about the underlying shocks through the resultant effect of those shocks on objects they follow. In particular, households tend to be relatively more informed about supply shocks than demand shocks, as supply shocks (which lead to negative comovement in output and inflation) most affect real income.[7] Firms, by contrast, tend to be relatively more informed about demand shocks than supply shocks, as demand shocks (which lead to positive comovement in output and

[4]Formally, I model the cost of attention as the Shannon mutual information times a scaling parameter, following Sims (2003). More precise (less noisy) signals are therefore more costly.

[5]This assumption is standard in this literature (see for example Woodford (2003); Mankiw and Reis (2002); Maćkowiak and Wiederholt (2009)), and it is not essential for the model results.

[6]The simple model assumes hand-to-mouth households and iid shocks, stripping out the intertemporal choice and the persistence of the full model.

[7]The idea is that a negative supply shock reduces output but increases inflation, resulting in a large decline in real income. A negative demand shock lowers both output and inflation, so the price decline partly offsets the income loss and real income is comparatively insulated.

inflation) have a greater impact on nominal marginal costs. I show that under a wide array of assumptions regarding preferences and model dynamics, the model can jointly generate households' supply-side view and firms' demand-side view (Proposition 3 and Corollary 1).

I then turn to the *"how much"*. In partial equilibrium, households and firms would pay more attention when the variable they choose to learn about is more volatile. This feature is common in rational inattention models (see, e.g., Sims (2003); Maćkowiak and Wiederholt (2009) and a large follow-up literature). A novel interaction arises in general equilibrium (Proposition 4): the attention efforts of households and firms are substitutes for demand shocks (households pay less attention if firms pay more attention), and complements for supply shocks (households pay less attention if firms pay less).[8] These interactions arise as the variables agents choose to learn about are endogenous and depend on others' attention and behavior.

While the above predictions on agents' views are derived in the simple model, I show that they can be generalized to a dynamic economy (see Proposition 6). In the dynamic model, agents' problems depend on persistent states and inherited beliefs, not just the current shocks. While with these dynamics it is hard to pin down the exact sign analytically, the same intuition discussed above carries over: if the variables that agents learn are more affected by supply shocks than demand shocks in equilibrium, then agents are more informed about supply shocks, which leads to a negative association between output and inflation, and vice versa.

Testing the mechanism. I solve the full model numerically and calibrate to U.S. data. The calibration is standard. Most of the parameters are estimated outside of the model, following the literature. Two parameters are particularly important, i.e., the households and firms marginal information costs. I calibrate those two parameters to match the correlation of each group's survey inflation expectations and realized inflation.[9]

The calibrated model is evaluated against two sets of *untargeted* evidence. First, it reproduces the opposing relationships between expected inflation and expected output growth for households, firms and professional forecasters (see Figure 5). By emphasizing the attention channel, the model generates the negative association by households, and the positive association by firms. Professional forecasters, who are assumed to be the full-information benchmark in the model, reflect the objective equilibrium correlation between output and inflation. Thus, the quantitative model jointly matches the different views by households and firms, as well as the equilibrium relationship, even though these are not targeted.

Second, the model makes a sharp prediction about how households' and firms' expectations of inflation and output respond to identified supply and demand shocks. In the model, as house-

[8]The strategic interactions in information acquisition have been studied by Maćkowiak and Wiederholt (2009) and Hellwig and Veldkamp (2009), among others, who argue that complementarity (substitutability) in information choices arises from complementarity (substitutability) in actions. Here I show that complementarity (substitutability) can also arise through the value of information in a general equilibrium model with multiple inattentive agents.

[9]Household inflation expectations are from the Michigan Survey of Consumers and firm inflation expectations are from the Livingston Survey. In the data, the correlation with realized inflation is about 0.3 for households and about 0.7 for firms.

holds and firms learn only endogenous variables, not exogenous shocks, they cannot fully distinguish the origin of shocks. This generates misattribution, similar to that in Lorenzoni (2009). Here, however, this misattribution differs systematically across households and firms. Specifically, since households' signals are relatively informative about supply shocks, an expansionary demand shock initially induces supply-like belief revisions: households raise their output expectations but lower their inflation expectations. Conversely, because firms' signals are relatively informative about demand shocks, a favorable supply shock induces demand-like belief revisions: firms revise expected output and inflation in the same direction. This provides a new empirical strategy: the IRFs of agents' average forecast of output and inflation to different aggregate shocks.[10] The estimated IRFs do show the misattribution, in the exact direction predicted by the model (Figures 6 and 7). This is a main empirical contribution of this paper and a "smoking gun" evidence of the asymmetric attention of households and firms.

Implications for business-cycle fluctuations. Endogenous attention has broader implications for aggregate dynamics. Because the *value* and *composition* of information depend on monetary policy and the volatility of shocks, the transmission of shocks varies across policy regimes and economic environments. I apply this mechanism to changes in the slope of the Phillips curve, in the spirit of Maćkowiak and Wiederholt (2015) and Afrouzi and Yang (2021). The key difference is that rational inattention operates on both sides of the economy. First, consistent with their results, the model implies that the shift toward more hawkish monetary policy in the post-Volcker period flattens the Phillips curve. More aggressive inflation stabilization reduces firms' attention, so they adjust prices less strongly to aggregate conditions, reducing the sensitivity of inflation to real activity. Household attention also adjusts accordingly, but the decline in firm attention is the primary force behind the flattening, making this result closely related to the existing firm-inattention mechanism.

The novel result concerns the post-pandemic steepening of the Phillips curve. Motivated by the large supply and demand shocks of the post-pandemic period, I increase the volatility of both shocks. Firms then acquire more information and adjust prices more strongly to aggregate conditions. Their greater attention also changes the equilibrium behavior of real income and real returns, shifting households' optimal information toward supply shocks. This reallocation dampens output-gap variation and steepens the Phillips curve, as observed in the data.[11] Both sides of attention are essential for this result. In a counterfactual in which firm attention responds to the higher-volatility environment but household information is held fixed, the Phillips curve becomes flatter rather than steeper. Two-sided endogenous attention can therefore provide a unified account of both the post-Volcker flattening and the post-pandemic steepening of the Phillips curve.

[10] To be consistent with the model specification, I use monetary policy shocks identified by Nakamura and Steinsson (2018) as demand shocks, and technology shocks identified through long-run restrictions following Galí (1999) as supply shocks.

[11] See Hobijn et al. (2023), Furlanetto and Lepetit (2024), Gelain and Lopez (2024), and Cerrato and Gitti (2022) for evidence on the post-pandemic steepening of the Phillips curve.

Implications for communication policy. The model has important implications for central-bank communication. Standard policy exercises often summarize an announcement by the expectation it is intended to move, such as expected inflation or the expected future policy rate. But moving one expectation generally changes beliefs about other variables as well, and these accompanying revisions depend on agents' information structures. I model communication as news that enters households' and firms' own signals. The announcement is normalized to move the same targeted expectation by the same amount for both groups, allowing me to isolate how heterogeneous information changes their interpretation of the message.

Consider first communication that raises expected future inflation. Households interpret the higher inflation primarily as evidence of an adverse supply shock and revise expected output downward. Firms interpret it primarily as evidence of stronger demand and revise expected output upward. Firms raise prices, while households reduce spending. The resulting output response is contractionary, in contrast to the standard full-information logic under which higher expected inflation lowers the expected real interest rate and stimulates current demand. Moreover, firms' pricing response further amplifies the initial contraction faced by households.

Communication of a lower future interest rate produces a different, but again heterogeneous, set of inferences. Households interpret the lower rate as the policy response to favorable supply shock, revising expected output upward and inflation downward. Firms interpret it as an expansionary demand shock and revise both expected output and expected inflation upward. Thus, even though the announcement moves the same targeted interest-rate expectation for both groups, it causes their inflation expectations to move in opposite directions. These experiments show that communication cannot be evaluated from its effect on the targeted expectation alone. Its aggregate consequences depend on how different agents interpret the broader economic information that the announcement appears to contain.

Related literature. This study contributes to the research agenda that seeks to develop a data-consistent model of expectation formation. Three closely related studies are Bhandari et al. (2024), Kamdar and Ray (2025) and Han (2024). The first two study only the consumer facet of the evidence, attributing the negative perceived relationship between inflation and real activity to optimal information compression or distorted beliefs. Han (2024) covers all agents but generates the opposing views from exogenously assigned information sets. Here, by contrast, the information differences arise endogenously as households and firms choose to learn about different objects given their distinct decision problems, and general equilibrium determines how their signals load on supply and demand shocks. This paper also moves beyond the unconditional correlations that motivate this literature and documents a new empirical fact, the different belief responses of households and firms to identified structural shocks, which can discriminate among theories. This relates to other work that also looks at impulse responses to provide richer information to discipline expectations, such as Angeletos et al. (2021).

This paper broadly relates to the rational inattention literature following Sims (2003). The core premise of this literature is that incentives drive attention, implying that agents pay more attention to more volatile and more important variables (e.g., Maćkowiak and Wiederholt (2009); Kohlhas and Walther (2021); Flynn and Sastry (2024)). Most of this literature considers one-sided rational inattention and therefore abstracts from the interaction between different groups' attention choices. The closest precedent is Maćkowiak and Wiederholt (2015), who also model two-sided rational inattention with inattentive firms and households. The contribution of this paper is to characterize how the two groups' attention is jointly determined in equilibrium, and in particular the strategic interaction between them through their best responses. Moreover, unlike existing mechanisms in which strategic information acquisition inherits complementarity or substitutability from actions (Hellwig and Veldkamp, 2009; Maćkowiak and Wiederholt, 2009), the strategic interaction here arises through the value of information in a general equilibrium model.

This paper also connects to a vast literature in macroeconomics on the role of imperfect information in business cycle dynamics (Lucas (1972); Woodford (2001); Eusepi and Preston (2010); Blanchard et al. (2013); Angeletos and La'o (2013); Chahrour and Ulbricht (2023); Afrouzi and Yang (2021) among others). Maćkowiak and Wiederholt (2015) and Afrouzi and Yang (2021) show that endogenous attention leads to state-dependent shock transmission, which can explain the flattening of the Phillips curve in the post-Volcker period. This paper in addition shows that the endogenous attention adjustment can also help explain the steepening of the Phillips curve and also highlights the role of the strategic interaction in driving such results.

Finally, the paper contributes to the analysis of policy and communication under imperfect information (Morris and Shin, 2005; Amador and Weill, 2010; Paciello, 2012). A recurring theme is that forward guidance operates differently once common knowledge fails or beliefs are heterogeneous (Angeletos and Lian, 2018; Andrade et al., 2019; Angeletos and Sastry, 2021; Coibion et al., 2023). In this paper, I show that households and firms interpret the same announcement through different information structures, communication that moves a targeted expectation can move their broader beliefs about output and inflation in opposite directions and undercut its intended effect.

Layout. In Section 2, I provide a closed-form characterization of households' and firms' attention choices under rational inattention in the illustrative model, and show that the resulting characterization of their views generalizes to a dynamic economy. In Section 3, I move to the full dynamic general equilibrium model and calibrate it. In Section 4, I confront the calibrated model with untargeted survey evidence, showing that it reproduces both the cross-sectional disagreement and the belief responses to identified shocks. In Section 5, I show that the model jointly explains the flattening of the Phillips curve in the post-Volcker period and its steepening in the post-pandemic period. In Section 6, I discuss the implications for communication. Section 7 concludes.

2 A Simple Model of Attention Choice

2.1 Environment

Households. There is a continuum of hand-to-mouth households indexed by $i \in [0,1]$. Household i in each period chooses consumption $C_{i,t}$ to maximize its expected utility and supplies labor $L_{i,t}$ such that the budget constraint binds. Household i's period utility at time t is

$$U\left(C_{i,t}, L_{i,t}\right) = \frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \frac{L_{i,t}^{1+\eta}}{1+\eta} \tag{2.1}$$

$$s.t.\ P_t C_{i,t} = W_t L_{i,t}, \quad C_{i,t} = \left[\int_0^1 C_{i,j,t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{2.2}$$

where $C_{i,j,t}$ is household i's demand for variety j given its price $P_{j,t}$ and $C_{i,t}$ is the final consumption good aggregated with a constant elasticity of substitution $\theta > 1$ across varieties. W_t is the nominal wage, and $P_t = [\int_0^1 P_{j,t}^{1-\theta} dj]^{1/(1-\theta)}$ is the aggregate price index. The parameter $\gamma > 1$ is the risk aversion coefficient and the parameter η is the inverse of the Frisch elasticity of labor supply. Households are hand-to-mouth here, so the simple model has no saving margin. The quantitative model in Section 3 adds bond trading and an intertemporal consumption choice.

Firms. There is a continuum of firms producing differentiated goods, each indexed by $j \in [0,1]$. Each firm j is a monopoly producer of its own variety and faces a demand curve $Y_{j,t} = (P_{j,t}/P_t)^{-\theta} Y_t$, where $Y_t = \int_0^1 Y_{j,t} dj$ is the aggregate output. Firm j hires labor $L_{j,t}$, pays wages W_t per worker, and produces with a linear technology $Y_{j,t} = A_t L_{j,t}$, where A_t is the aggregate productivity.

In each period, firm j sets the price $P_{j,t}$ for its own product to maximize its expected profit and produces a sufficient quantity of goods to meet the demand $Y_{j,t}$. Prices are fully flexible (nonetheless, as discussed later in Section 2.4, price stickiness arises endogenously from firms' inattention). The profit of firm j at time t, discounted by the household's marginal utility of consumption, is expressed as

$$\Pi_{j,t}(P_{j,t}, L_{j,t}, Y_{j,t}) = \frac{1}{P_t C_t^{\gamma}} \left[P_{j,t} Y_{j,t} - (1-\theta^{-1}) W_t L_{j,t}\right] \tag{2.3}$$

where $(1-\theta^{-1})W_t$ denotes the subsidized wage rate, with the subsidy θ^{-1} paid to eliminate steady-state distortions introduced by monopolistic competition.

Central bank. For analytical tractability, I assume that the central bank directly controls the nominal aggregate demand $Q_t \equiv P_t Y_t$. This assumption allows for a closed-form characterization of the solution.[12] I consider a more standard Taylor rule in the quantitative model in Section 3. I

[12] Assuming that the monetary authority directly controls the nominal aggregate demand is a popular framework in the rational inattention literature to study the effects of monetary policy on pricing. See for example Mankiw et al. (2003); Woodford (2003); Maćkowiak and Wiederholt (2009); Paciello (2012); Afrouzi and Yang (2021) among others.

further assume that the central bank has full information and interpret it as the model counterpart of the professional forecasters in the survey.

Shocks. The economy is subject to both demand and supply shocks. I model the demand shock as a shock to the nominal aggregate demand ($q_t \equiv \log Q_t$), and the supply shock as a shock to all firms' productivity levels ($a_t \equiv \log A_t$). The two exogenous processes are Gaussian white noise with variances $\sigma_q^2 > 0$ and $\sigma_a^2 > 0$, and are mutually independent.

2.2 Attention costs and information structure

Costly attention. In this environment, agents must pay attention in order to be aware of the economic conditions. While the cost of attention can, in principle, take many different forms (see e.g., Hébert and Woodford (2018)), I follow Sims (2003) and model the attention costs as linear in Shannon's mutual information $\mu\mathcal{I}\left(X; S^t|S^{t-1}\right)$, where μ is the marginal cost of attention. Specifically, $s_t \in \mathcal{S}^t$ denotes the signals at time t, and $\mathcal{S}^t$ is the set of available signals. The history of signals up to time t is denoted by $S^t = S^{t-1} \cup s_t$. Mutual information is defined as

$$\mathcal{I}\left(X; S^t|S^{t-1}\right) \equiv h(X|S^{t-1}) - \mathbb{E}\left[h(X|S^t)|S^{t-1}\right]$$

This measures the reduction in entropy of the object X due to information gained from signal S^t conditional on the history of signals S^{t-1}.

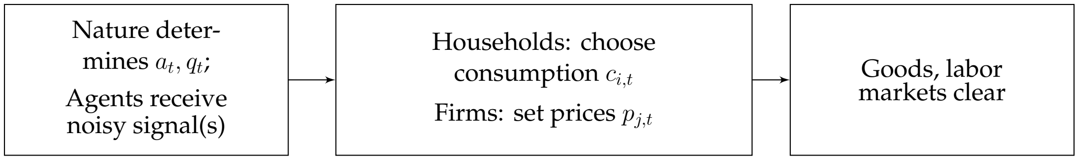
This formulation assumes that agents do not forget information over time and thus the information chosen today can have a continuation value. In the simple model presented in this section, this condition does not matter as shocks are i.i.d., so the knowledge about the shocks today does not affect future priors. However, in the full model presented in Section 3, where shock processes are more complex and intertemporal decisions are involved, past information becomes useful.

Information structure. Following Sims (2003) and Maćkowiak et al. (2018) and many others, I allow agents to design their signals flexibly. Under optimal signal design, agents choose signals about the payoff-relevant objects for their own decisions.[13] A nice feature of this assumption is that the model-implied optimal signals are consistent with survey evidence on households' and firms' attention choices, which I discuss in Section 4.3.

Timing. In the initial period $t = 0$, households and firms make their ex ante attention choices, which determine the form and precision of the associated signals. In each subsequent period $t > 0$, shocks (q_t, a_t) realize. The economy proceeds through three stages: (i) depending on their respective attention choices, households and firms receive different forms of signals with different precision levels; (ii) based on their respective signals, households choose their consumption and

[13] Each agent chooses a signal about the object relevant to its own decision, and its informativeness about the individual shocks is a derived object. This is natural in the present context, where agents do not directly observe the underlying shocks.

firms set their prices for their own varieties; (iii) after their choices are committed, households supply labor to cover their consumption and firms produce sufficient goods to meet the demand. Finally, the real wage adjusts to clear the labor market.



2.3 Attention problems of households and firms

To tractably study equilibrium, I simplify the households' and firms' objectives with quadratic approximations. Both approximations are derived in Appendix B.1. Throughout, lowercase letters denote log deviations from the steady state.

Households. After the approximation, household i's objective (2.1) at time t can be expressed as the utility loss from deviating from the optimal consumption level $c^*_{i,t}$ – the consumption level that households would choose under full information [14]

$$-\frac{\gamma+\eta}{2}\left(c_{i,t}-c^*_{i,t}\right)^2 + \text{terms independent of } c_{i,t} \tag{2.4}$$

Here $c_{i,t}$ is the actual consumption choice made by household i based on its information set. When the actual consumption deviates from its optimal level, there is a utility loss, as captured by the quadratic difference. In particular, the utility loss increases with the risk aversion coefficient γ and the inverse of the Frisch elasticity of labor supply η.

The optimal consumption under full information is obtained by equating the marginal rate of substitution between consumption and leisure to the real wage[15]

$$c^*_{i,t} = \lambda\left(w_t - p_t\right), \qquad \lambda \equiv \frac{1+\eta}{\gamma+\eta} \tag{2.5}$$

The equation states that optimal consumption is a function of the real wage. Thus, the payoff-relevant unknown for households is the *real wage*.

Substituting the optimal consumption from Equation (2.5) into the loss function (2.4) and adding the attention cost yields household i's attention problem:

$$\max_{\{s_{i,t}\in\mathcal{S}^t_h\}} \mathbb{E}^h_t\left[-\frac{\gamma+\eta}{2}\left(c_{i,t}-\lambda\left(w_t-p_t\right)\right)^2 - \mu^h\mathcal{I}\left(w_t-p_t; s_{i,t}\right)\right] \tag{2.6}$$

[14]The first-order term of this approximation drops out due to the envelope theorem: there are no first-order costs of deviating from $c^*_{i,t}$. See Appendix B.1 for the full derivation.

[15]The optimal consumption is derived by substituting out $l_{i,t}$ using the budget constraint $p_t + c_{i,t} = w_t + l_{i,t}$ in the intra-temporal optimality condition $\gamma c_{i,t} + \eta l_{i,t} = w_t - p_t$.

The first term is the benefit of attention, more prices signal brings $c_{i,t}$ closer to the optimal level. The second term is its cost, which is the marginal cost of attention $\mu^h > 0$ times the expected reduction in the entropy of the real wage from observing the signal $s_{i,t}$.

Firms. Symmetrically, firm j's objective becomes the profit loss from deviating from the full-information price $p^*_{j,t}$,

$$-\frac{\theta-1}{2}\left(p_{j,t}-p^*_{j,t}\right)^2 + \text{terms independent of } p_{j,t} \tag{2.7}$$

The loss scales with $\theta - 1$, so firms with more elastic demand (higher θ) experience larger profit losses when charging a suboptimal price. The firm's optimal price under full information is its nominal marginal cost

$$p^*_{j,t} = w_t - a_t \tag{2.8}$$

Thus the payoff-relevant unknown for firms is their *nominal marginal cost*. Adding the attention cost yields firm j's attention problem:

$$\max_{\{s_{j,t}\in\mathcal{S}^t_f\}} \mathbb{E}^f_t\left[-\frac{\theta-1}{2}\left(p_{j,t}-(w_t-a_t)\right)^2 - \mu^f \mathcal{I}\left(w_t-a_t; s_{j,t}\right)\right] \tag{2.9}$$

The first term is the benefit of attention, it brings $p_{j,t}$ closer to the optimal level, the nominal marginal cost. The second term is its cost: the marginal cost of attention $\mu^f > 0$ times the expected reduction in the entropy of the nominal marginal cost from observing the signal $s_{j,t}$.

Equilibrium. The equilibrium of the model is defined as in Definition 1.

Definition 1 (Equilibrium)**.** *Given the processes for the aggregate demand and productivity shocks $\{q_t, a_t\}_{t\geq 0}$, a general equilibrium of this economy is an allocation for every household $i \in [0,1]$, $\Omega_i \equiv \{s_{i,t} \in \mathcal{S}_{i,t}, C_{i,t}, L_{i,t}\}^\infty_{t=0}$, given their initial set of signals; an allocation for every firm $j \in [0,1]$, $\Omega_j \equiv \{s_{j,t} \in \mathcal{S}_{j,t}, P_{j,t}, L_{j,t}, Y_{j,t}\}^\infty_{t=0}$ given their initial set of signals; a set of prices $\{P_t, W_t\}^\infty_{t=0}$, such that*

1. *Given the processes for $\{P_t, W_t\}^\infty_{t=0}$ and all firms' decisions $\{\Omega_j\}_{j\in[0,1]}$, every household i's allocation solves the attention problem (2.6);*
2. *Given the processes for $\{P_t, W_t\}^\infty_{t=0}$ and all households' allocations $\{\Omega_i\}_{i\in[0,1]}$, every firm j's allocation solves the attention problem (2.9);*
3. *The equilibrium processes $\{P_t, W_t\}^\infty_{t=0}$ are consistent with the allocations of households and firms, $\{\Omega_i\}_{i\in[0,1]}$ and $\{\Omega_j\}_{j\in[0,1]}$.*

To proceed, Section 2.4 solves households' and firms' attention choices in partial equilibrium, and Section 2.5 determines them in general equilibrium. Section 2.6 then shows how heterogeneous attention across households and firms accounts for the heterogeneous views observed in survey data. Because the model features two-sided rational inattention, the attention allocations of households and firms interact strategically in equilibrium, which I discuss in Section 2.7.

2.4 Solving the attention problem

I first solve each agent's problem in partial equilibrium, taking the equilibrium wage and price processes as given. Section 2.5 then determines them in general equilibrium.

Both (2.6) and (2.9) are standard linear-quadratic-Gaussian (LQG) rational-inattention problems: a quadratic loss in the gap between the action and a Gaussian target, with a Shannon-information cost. Their solution is well known (Sims, 2003; Maćkowiak and Wiederholt, 2009; Maćkowiak et al., 2018): the optimal signal is a single noisy observation of the target, the action is the posterior mean, and attention is summarized by a single Kalman gain.

Proposition 1 (Signals and attention). *Let* $\sigma_\omega^2 \equiv \mathrm{Var}(w_t - p_t)$ *and* $\sigma_{mc}^2 \equiv \mathrm{Var}(w_t - a_t)$ *denote the (equilibrium) volatilities of the two targets. Taking these volatilities as given:*

1. *Households and firms each acquire a single noisy signal about their respective payoff-relevant unknowns,*

$$s_{i,t} = (w_t - p_t) + e_{i,t}, \qquad s_{j,t} = (w_t - a_t) + e_{j,t},$$

where $e_{i,t}$ *and* $e_{j,t}$ *are mean-zero noise terms, independent of the targets and across agents.*

2. *The optimal attention levels, measured by Kalman gains, are*

$$\xi_h = \max\left(0,\, 1 - \frac{\mu^h}{(\gamma+\eta)\lambda^2\,\sigma_\omega^2}\right), \qquad \xi_f = \max\left(0,\, 1 - \frac{\mu^f}{(\theta-1)\,\sigma_{mc}^2}\right). \tag{2.10}$$

3. *Each agent sets its action equal to the posterior mean of its optimal action given its signal, so the induced actions are* $c_{i,t} = \lambda\,\xi_h(w_t - p_t + e_{i,t})$ *and* $p_{j,t} = \xi_f(w_t - a_t + e_{j,t})$.

Proof. See Appendix B.2.

Two points are worth emphasizing. Part 1 of Proposition 1 shows that households and firms optimally choose to acquire different information. In particular, households learn about the real wage $w_t - p_t$, firms about the nominal marginal cost $w_t - a_t$. Thus, their signals contain different information about the underlying economy. These model-implied attention choices are consistent with the survey evidence in Section 4.3, where households report attending mainly to employment-related news and firms to their nominal labor costs.

Part 2 of Proposition 1 captures the optimal attention level. Here ξ_h and ξ_f are between 0 and 1, with 0 corresponding to no attention and 1 to full attention. Agent k pays more attention when (i) their objectives are more sensitive to inattention mistakes (curvature $(\gamma+\eta)\lambda^2$ for households, $\theta-1$ for firms), and (ii) attention is less costly (low μ^k); (iii) targets are more volatile (σ_ω^2 for households and σ_{mc}^2 for firms). Importantly, these volatilities are determined endogenously in equilibrium, and depend on households' and firms' attention choices and decisions. For example, the volatility of real wage largely depends on firms' pricing decisions, which in turn depend on their attention to the nominal marginal cost. Similarly, the volatility of the nominal marginal cost depends on households' consumption decisions, which depend on their attention to the real wage.

Making these dependencies mutually consistent closes the model, which I take up in Section 2.5. Their interdependence also makes attention *strategic*: because each agent's attention shapes the volatility of the other's target – and hence the value of the other's information – households' and firms' attention choices are determined jointly rather than in isolation. I return to this in Section 2.7.

2.5 Closing the model: equilibrium attention

Section 2.4 characterizes households' and firms' attention choices taking their respective target processes as given. Households acquire information about the real wage, while firms acquire information about the nominal marginal cost. These variables, however, are determined in equilibrium. This section closes the model and derives the equilibrium.

Proposition 1 gives the household's consumption choice and the firm's price. Let $c_t \equiv \int_0^1 c_{i,t}\, di$ and $p_t \equiv \int_0^1 p_{j,t}\, dj$ denote aggregate consumption and the aggregate price index. As the idiosyncratic signal noise is independently distributed across agents, it averages out in the aggregate. Aggregate consumption and price are given by

$$c_t = \lambda\, \xi_h\, (w_t - p_t), \qquad p_t = \xi_f\, (w_t - a_t). \tag{2.11}$$

Define $D \equiv \xi_f + \lambda\, \xi_h\, (1 - \xi_f)$. Solving the market-clearing conditions gives the two equilibrium targets as explicit functions of the shocks,

$$\underbrace{w_t - p_t}_{\text{household target}} = \frac{1-\xi_f}{D}\, q_t + \frac{\xi_f}{D}\, a_t, \qquad \underbrace{w_t - a_t}_{\text{firm target}} = \frac{1}{D}\, q_t - \frac{\lambda\, \xi_h}{D}\, a_t. \tag{2.12}$$

The targets (2.12) imply the equilibrium volatilities $\sigma_\omega^2 = \left[(1-\xi_f)^2\sigma_q^2 + \xi_f^2\sigma_a^2\right]/D^2$ and $\sigma_{mc}^2 = \left[\sigma_q^2 + \lambda^2\xi_h^2\sigma_a^2\right]/D^2$. Substituting these into the individual solution (2.10) makes each attention level a function of the other, giving the fixed point below.

Proposition 2 (Equilibrium attention)**.** *Suppose both shocks have positive variance and consider an interior equilibrium with $\xi_h, \xi_f > 0$.*[16]

1. *Households acquire a single signal about the equilibrium real wage,*

$$s_{i,t} = b_{h,q} q_t + b_{h,a} a_t + e_{i,t}, \quad \textit{with} \quad b_{h,q} = \frac{1-\xi_f}{D}, \quad b_{h,a} = \frac{\xi_f}{D} \tag{2.13}$$

[16]A fixed point exists, and the interior equilibrium – in which both agents acquire strictly positive attention – is unique: the attention best-response map is a contraction when the general-equilibrium feedback is not too strong (Appendix B.3). Any multiplicity is confined to corner solutions ($\xi_k = 0$), which arise when attention is sufficiently costly relative to the volatility of the target, as captured by the max operator in (2.10).

2. *Firms acquire a single signal about equilibrium nominal marginal cost,*

$$s_{j,t} = b_{f,q} q_t + b_{f,a} a_t + e_{j,t}, \quad \textit{with} \quad b_{f,q} = \frac{1}{D}, \quad b_{f,a} = -\frac{\lambda\,\xi_h}{D} \tag{2.14}$$

3. *The attention levels (ξ_h, ξ_f) jointly solve the fixed point*

$$1 - \xi_h = \frac{\mu^h D^2}{(\gamma+\eta)\lambda^2\Big[(1-\xi_f)^2\sigma_q^2 + \xi_f^2\sigma_a^2\Big]}, \qquad 1 - \xi_f = \frac{\mu^f D^2}{(\theta-1)\big[\sigma_q^2 + \lambda^2\xi_h^2\sigma_a^2\big]}. \tag{2.15}$$

Proof. See Appendix B.3.

Two features of this equilibrium matter for what follows. First, the household and firm signals (2.13)–(2.14) load differently on the two shocks, so they carry asymmetric information about them. Second, the two attention levels are jointly determined by the fixed point (2.15). The first feature drives the heterogeneous beliefs of Section 2.6; the second makes attention allocation strategic, which I take up in Section 2.7.

Information content of the two signals. Before turning to how these asymmetric signals generate heterogeneous views, it is useful to examine their information content. As (2.13)–(2.14) show, each signal is informative about both shocks. A signal carries relatively more information about one shock than the other when (i) that shock is more volatile (high σ_q^2 or σ_a^2), or (ii) the signal loads more heavily on it (high $|b_{k,q}|$ or $|b_{k,a}|$). This is the familiar principle that attention gravitates toward variables that are both important and volatile (Maćkowiak and Wiederholt, 2009; Kohlhas and Walther, 2021).

Consider the loadings first. By (2.13), the household's signal loads relatively more on supply shocks when firms are more attentive.[17] The reason lies in firms' pricing. A demand shock raises the nominal wage, and attentive firms raise their prices in step, so wage and price move up together and largely cancel in the real wage $w_t - p_t$. A supply shock instead raises the nominal wage *and* lowers prices, wage and price now move in opposite directions, leads to large variation in real wage. The real wage is therefore more responsive to supply than to demand shocks, and the household's signal loads more on supply – the more so the more attentive are firms. In the limit $\xi_f \to 1$ the demand response cancels entirely and the real wage loads on supply alone, $w_t - p_t = a_t$, the classical dichotomy.

By contrast, (2.14) shows that the firm's signal loads relatively more on demand shocks: its demand loading $1/D$ exceeds its supply loading $\lambda\xi_h/D$ in absolute value whenever $\gamma > 1$ (so that $\lambda < 1$). Intuitively, following a positive supply shock the nominal marginal cost $mc_t = w_t - a_t$ falls on impact, and firms cut prices. Lower prices raise demand, and with $\gamma > 1$ the income effect dominates, so households reduce their labor supply and the wage rises. The wage

[17] The supply loading exceeds the demand loading when $\xi_f > 0.5$; an empirical estimate based on the correlation between inflation and firms' expected inflation puts ξ_f at approximately 0.65.

response offsets the initial fall in the nominal marginal cost, which therefore responds only weakly to supply shocks. A demand shock meets no such offset. The nominal marginal cost is thus more responsive to demand than to supply shocks, and the firm's signal loads more on demand. The supply loading is dampened further when households pay less attention (lower ξ_h).

In sum, the signal loadings have clear predictions. The household's signal loads more on supply shocks, as the real wage responds more to supply than to demand shocks due to firms' pricing behavior, and the firm's signal loads more on demand shocks, as the nominal marginal cost responds more to demand than to supply shocks due to the income effect.

The loadings capture how each target responds to a unit change in a shock, but the volatility of the shocks also matters: a signal carries more information about the shock that is more volatile. Consider two extreme cases. If the supply shock is very volatile, $\sigma_a^2 \gg \sigma_q^2$, both signals are dominated by supply, so both agents are informed mainly about supply shocks. If instead the demand shock is very volatile, $\sigma_q^2 \gg \sigma_a^2$, both signals are dominated by demand shocks.

With comparable shock volatilities, these loadings make the household's signal more informative about supply shocks and the firm's about demand shocks. When the volatilities differ sharply, they temper or overturn this tilt. Figure 2 shows the relative informativeness of each signal, supply over demand, as the relative shock volatility varies. The shaded band marks the region where the two signals are asymmetrically informative about different shocks: the household's signal is more informative about supply shocks, the firm's about demand shocks.

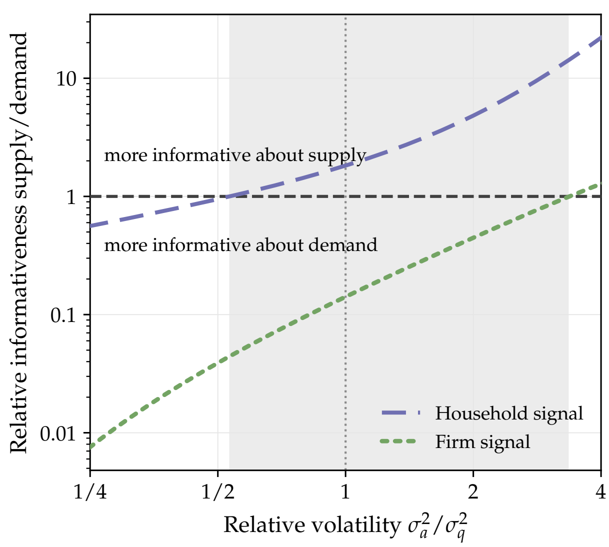


Figure 2: Relative informativeness of the two signals, supply over demand

Notes: Each curve plots the supply-to-demand variance share of a signal, $b_{k,a}^2\sigma_a^2/(b_{k,q}^2\sigma_q^2)$, for the household ($k = h$, dashed) and firm ($k = f$, dotted); above the horizontal line a signal is more informative about supply, below it about demand, and the shaded band marks where the two are asymmetrically informed.

2.6 Who holds which view

I now show that these different signals generate the opposing views in the data. Write equilibrium output and prices as

$$y_t = \Psi_{y,q} q_t + \Psi_{y,a} a_t, \qquad p_t = \Psi_{p,q} q_t - \Psi_{p,a} a_t, \tag{2.16}$$

Here the Ψ's denote the responses of aggregate output y_t and the aggregate price p_t to the demand and supply shocks. Their specific values are determined endogenously by the equilibrium attention choices and decisions of firms and households, and are not central to the analysis in this section. A positive demand shock is expansionary and inflationary ($\Psi_{y,q}, \Psi_{p,q} > 0$), while a supply shock raises output but lowers prices ($\Psi_{y,a}, \Psi_{p,a} > 0$). The equilibrium covariance, equivalently, the covariance perceived by a full-information agent, is then

$$\mathrm{Cov}(y_t, p_t) = \Psi_{y,q} \Psi_{p,q}\, \sigma_q^2 - \Psi_{y,a} \Psi_{p,a}\, \sigma_a^2. \tag{2.17}$$

It can be positive or negative, depending on the equilibrium responses Ψ and the shock volatilities σ_q^2, σ_a^2. For example, if the variation in the economy is dominated by supply shocks, then the covariance is negative, and if dominated by demand shocks, it is positive.

Proposition 3 (Perceived output–price covariance)**.** *Let agent $k \in \{h, f\}$ observe the single signal $s_{k,t} = b_{k,q} q_t + b_{k,a} a_t + e_{k,t}$. Then*

$$\mathrm{Cov}\Big(\mathbb{E}^k[y_t], \mathbb{E}^k[p_t]\Big) = \frac{\Lambda_y^k \, \Lambda_p^k}{\mathrm{Var}(s_{k,t})}, \qquad \begin{aligned} \Lambda_y^k &\equiv \Psi_{y,q} b_{k,q} \sigma_q^2 + \Psi_{y,a} b_{k,a} \sigma_a^2, \\ \Lambda_p^k &\equiv \Psi_{p,q} b_{k,q} \sigma_q^2 - \Psi_{p,a} b_{k,a} \sigma_a^2. \end{aligned} \tag{2.18}$$

Since $\mathrm{Var}(s_{k,t}) > 0$, the sign of the perceived comovement is the sign of the product of the two projections: $\mathrm{sign}\big(\mathrm{Cov}\big) = \mathrm{sign}\big(\Lambda_y^k\big)\mathrm{sign}\big(\Lambda_p^k\big)$. The agent holds a demand-side view *($\mathrm{Cov} > 0$) when Λ_y^k and Λ_p^k share a sign, and a* supply-side view *($\mathrm{Cov} < 0$) when they differ.*

Proof. See Appendix B.6.

The perceived covariance takes a different form from the full-information one. Because the agent observes a single signal, its output forecast $\mathbb{E}^k[y_t]$ and price forecast $\mathbb{E}^k[p_t]$ are both proportional to that signal. Their covariance is therefore the product of the signal's covariance with output, Λ_y^k, and its covariance with price, Λ_p^k, scaled by the signal variance, so its sign is the sign of this product.

The sign of this product follows from which shock the signal is more informative about. Recall from (2.16) that a demand shock raises output and prices together, whereas a supply shock raises output but lowers prices. A signal more informative about demand therefore covaries positively with both output and prices, so Λ_y^k and Λ_p^k share a sign and the perceived covariance is positive. A signal more informative about supply, by contrast, covaries positively with output but negatively with prices, so Λ_y^k and Λ_p^k differ in sign and the covariance is negative. Which shock a signal is more informative about depends, in turn, on its loadings and the shock volatilities (Section 2.5).

Combined with the comparative static there, this yields the model's central prediction: households, whose signal is more informative about supply shocks, perceive a negative covariance, and thus hold a supply-side view. Firms, by contrast, whose signal is more informative about demand shocks, perceive a positive covariance, and thus hold a demand-side view. Substituting the equilibrium loadings from (2.13)–(2.14) into (2.18) delivers the exact range of relative volatility under which households hold a supply-side view and firms a demand-side view.

Corollary 1 (Heterogeneous views)**.** *If the relative volatility lies in the intermediate range*

$$\frac{\Psi_{p,q}\,(1-\xi_f)}{\Psi_{p,a}\,\xi_f} \;<\; \frac{\sigma_a^2}{\sigma_q^2} \;<\; \frac{\Psi_{y,q}}{\Psi_{y,a}\,\lambda\,\xi_h}, \tag{2.19}$$

then

1. *Households hold a supply-side view, as their signal is more informative about supply shocks.*
2. *Firms hold a demand-side view, as their signal is more informative about demand shocks.*
3. *Full-information agents' perceived covariance equals the equilibrium covariance* (2.17)*, which can be either positive or negative, depending on the equilibrium responses* Ψ *and the shock volatilities* σ_q^2, σ_a^2*.*

Corollary 1 confirms the intuition of the previous section: over an intermediate range of relative volatility the two agents hold opposing views, as in the survey data. Outside this range one shock dominates the other's volatility, so both signals are more informative about it and the agents agree. The simple model thus reproduces the contrasting survey views. To pin down the magnitude of the covariance, I extend it to a richer quantitative setting and solve it numerically in Section 3.

A caveat here is that the covariance in (2.18) is between current output and prices, whereas the survey measures the perceived correlation between output growth and inflation over the next year. In this static model, information acquired today is useless for forecasting, so expectations about future output and prices are always zero. The dynamic model of Section 3 delivers the object the survey measures, and I show that the same logic carries over.

2.7 Strategic interaction in attention

I now turn to the strategic interaction in attention allocation between households and firms. As shown in Section 2.5, the two attention levels (ξ_h, ξ_f) are determined jointly by the fixed point (2.15), so the two choices interact. The interaction runs through a single channel: each group's attention reshapes the magnitude of the other's equilibrium loadings.

From Equation (2.13), raising firm attention ξ_f lowers the household signal's demand loading and raises its supply loading,

$$b_{h,q} = \frac{1-\xi_f}{D}\ \downarrow, \qquad b_{h,a} = \frac{\xi_f}{D}\ \uparrow \qquad (\text{as } \xi_f \uparrow) \tag{2.20}$$

This is intuitive. Under a demand shock, more attentive firms track their marginal cost more closely, so prices move more with the nominal wage and the real wage responds less to demand. Under a supply shock, more attentive firms cut prices more strongly after a positive productivity shock, leading to larger variation in the real wage. Households therefore observe a signal that loads less on demand and more on supply when firms' attention rises. Household attention reshapes the firm's loadings symmetrically,

$$b_{f,q} = \frac{1}{D} \downarrow, \qquad |b_{f,a}| = \frac{\lambda \xi_h}{D} \uparrow \qquad (\text{as } \xi_h \uparrow) \tag{2.21}$$

These loading shifts affect attention through the volatility of the target. Since an agent's attention increases in the volatility of its target (2.10), and this volatility increases in the magnitude of the loadings, an increase in a loading raises the other group's attention and a decrease lowers it. I characterize the resulting interaction first for a single shock and then for both.

With a single shock, the target loads on that shock alone, so the sign of the attention response follows directly from the direction of the loading shift.

Proposition 4 (Single shock)**.** *With a single shock, household and firm attention are strategic substitutes under demand shocks and strategic complements under supply shocks,*

$$\textit{demand:} \quad \frac{\partial \xi_h}{\partial \xi_f} \leq 0, \qquad \qquad \textit{supply:} \quad \frac{\partial \xi_h}{\partial \xi_f} > 0,$$

and $\partial \xi_f / \partial \xi_h$ has the same sign in each case.

Proof. See Appendix B.4.

Under demand shocks alone, the household's target is $b_{h,q} q_t$. A rise in firm attention lowers $b_{h,q}$, making the target less volatile and reducing household attention; the two attention levels are strategic substitutes. Under supply shocks alone, the target is $b_{h,a} a_t$. A rise in firm attention raises $b_{h,a}$, making the target more volatile and raising household attention; the two attention levels are strategic complements. Figure 3 illustrates both cases.

With both shocks present, a rise in firm attention moves $b_{h,q}$ and $b_{h,a}$ in opposite directions. Its effect on the volatility of the real wage σ_ω^2 is the sum of the two effects and is ambiguous, so whether household attention rises or falls depends on which shock dominates. Its effect on the *composition* of the signal is not ambiguous: the loading ratio $b_{h,a}/b_{h,q} = \xi_f/(1 - \xi_f)$ increases in ξ_f, so the household signal tilts toward supply. Symmetrically, a rise in household attention increases $|b_{f,a}/b_{f,q}| = \lambda \xi_h$, tilting the firm signal toward supply as well. Proposition 5 summarizes these effects.

Proposition 5 (Both shocks)**.** *When both shocks are present, the effect of one group's attention on the other's attention* level *is ambiguous in sign, whereas its effect on the* composition *of the other's signal is not: greater attention by either group tilts both signals toward supply shocks relative to demand shocks.*

Proof. See Appendix B.5.

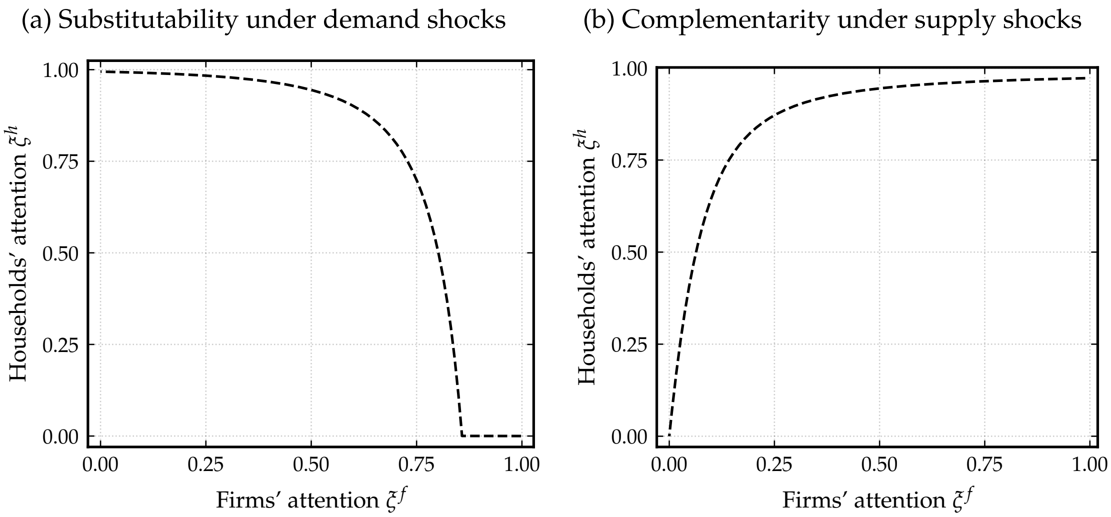


Figure 3: Strategic interactions in attention allocation

Notes: Attention (Kalman gain) of households and firms. μ^h is fixed and μ^f varies; lower μ^f raises firm attention. Household attention moves with firm attention.

In both cases, the interaction works through the equilibrium loading (and thus volatility) of the other's target. This channel differs from the strategic interactions in information acquisition considered by Maćkowiak and Wiederholt (2009) and Hellwig and Veldkamp (2009) among others, which argue that complementarity (substitutability) in information choices arises from the complementarity (substitutability) in actions. I demonstrate that complementarity (substitutability) can also arise through the value of information in a general equilibrium model with multiple inattentive agents.

The strategic interaction has implications for the business cycle. Suppose both shocks grow more volatile while their relative volatility is unchanged. Under exogenous attention, their relative importance in driving fluctuations is unchanged. Under endogenous attention, the higher volatility raises attention, and by Proposition 5, both household's and firm's signal tilt more to supply shocks. Supply shocks therefore come to drive a larger share of fluctuations. In Section 5 I show that this mechanism is important in explaining the post-pandemic steepening of the Phillips curve.

2.8 A general characterization of joint beliefs in the dynamic model

The simple model shows how agents come to hold different beliefs: their signals load differently on the shocks, so they are differently informed about them, and their beliefs reflect the information they hold.

This section shows that the logic carries to a general dynamic environment. In a dynamic environment, the main complication is that agents' targets depend not only on the contemporaneous shocks but also on persistent state variables and inherited beliefs. As a result, whether an agent holds a demand-side or supply-side view cannot in general be determined from the contemporaneous shock loadings alone. I first characterize the dynamic information problem faced by each group. I then construct a joint state-space representation for actual outcomes and subjective beliefs. Finally, I derive a sufficient statistic for the sign of the perceived output–price covariance.

Let $X_t \in \mathbb{R}^n$ denote the equilibrium state, collecting the predetermined endogenous variables and the exogenous shocks. Any linear rational-expectations equilibrium admits the representation

$$X_t = AX_{t-1} + Q\varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, I_m), \tag{2.22}$$

where $A \in \mathbb{R}^{n\times n}$ governs the propagation of the state and $Q \in \mathbb{R}^{n\times m}$ maps the structural innovations into it. The matrices A and Q are equilibrium objects and generally depend on the information choices of households and firms. The economy is driven by demand and supply shocks, $\varepsilon_t = (\varepsilon_t^d, \varepsilon_t^s)'$. These are not restricted to the two shocks of the simple model, and each of ε_t^d and ε_t^s may contain several shocks; nonetheless, all demand shocks move output and prices in the same direction, and all supply shocks move them in opposite directions.

An agent in group $k \in \{h, f\}$ chooses an action with a quadratic payoff, so its full-information optimal action is a linear combination of the state,

$$a_t^{k,*} = B_k X_t. \tag{2.23}$$

The row vector B_k maps the state into group k's optimal action; it is the dynamic counterpart of the signal loadings b of the simple model. For clarity I take each agent to choose a single action, so that it acquires one signal about its target; the characterization extends to a p-dimensional action (Appendix C). As in Section 2, the agent acquires a single Gaussian signal about this target,

$$s_{i,t}^k = B_k X_t + e_{i,t}^k, \qquad e_{i,t}^k \sim \mathcal{N}(0, R^k), \tag{2.24}$$

with noise variance R^k set by the attention problem (a lower R^k is more attention). The agent updates by the Kalman filter, forms the posterior mean $\hat{X}_{i,t}^k$, and acts on it, $a_{i,t}^k = B_k \hat{X}_{i,t}^k$.

Output and prices are linear in the state, $y_t = \Psi_y' X_t$ and $p_t = \Psi_p' X_t$, where Ψ_y and Ψ_p collect the responses of output and prices to each element of the state. The perceived output and prices for each group are given by $\mathbb{E}_t^k[y_t] = \Psi_y' \hat{X}_t^k$ and $\mathbb{E}_t^k[p_t] = \Psi_p' \hat{X}_t^k$. Stacking the state and beliefs into a joint linear system (Appendix C) yields a law of motion whose stationary covariance delivers every actual and perceived second moment. The object of interest is the covariance each group perceives between output and prices, $\mathrm{Cov}(\mathbb{E}_t^k[y_t], \mathbb{E}_t^k[p_t])$. Writing the prior state covariance as $\Sigma_{1|0}^k \equiv \mathrm{Var}(X_t \mid \mathcal{I}_{t-1}^k)$, this covariance is characterized in Proposition 6.

Proposition 6 (Perceived covariance in dynamic model)**.** *In a stationary linear-Gaussian equilibrium, group k's perceived output–price covariance is*

$$\mathrm{Cov}\Big(\mathbb{E}_t^k[y_t], \mathbb{E}_t^k[p_t]\Big) = \frac{d_k' \, \mathcal{W}_{yp} \, d_k}{S_k}, \qquad d_k \equiv \Sigma_{1|0}^k B_k', \quad S_k \equiv B_k \Sigma_{1|0}^k B_k' + R^k > 0, \tag{2.25}$$

where $\mathcal{W}_{yp}$ solves the Lyapunov equation $\mathcal{W}_{yp} = \Psi_{yp} + A'\mathcal{W}_{yp}A$ with $\Psi_{yp} \equiv \frac{1}{2}(\Psi_y\Psi_p' + \Psi_p\Psi_y')$. Since $S_k > 0$,

$$\operatorname{sign}\operatorname{Cov}\Big(\mathbb{E}_t^k[y_t], \mathbb{E}_t^k[p_t]\Big) = \operatorname{sign}\big(d_k'\,\mathcal{W}_{yp}\,d_k\big). \tag{2.26}$$

Proof. See Appendix C.

The three objects in (2.25) play distinct roles: B_k says which states matter for group k's decision and $\Sigma_{1|0}^k$ which states are most uncertain, so an agent attends to states that are both important and volatile. These are the dynamic counterparts of the objects in the simple model: B_k generalizes the signal loadings b, $\Sigma_{1|0}^k$ generalizes the shock volatilities (σ_q^2, σ_a^2). The matrix $\mathcal{W}_{yp}$ records the covariance between output and prices induced by each state in X_t, cumulated over how that state propagates. The comovement induced by state i, $e_i'\mathcal{W}_{yp}e_i$, is positive when the state moves output and prices in the same direction, as a demand shock does, and negative when it moves them in opposite directions, as a supply shock does. If d_k concentrates on supply-type directions, the quadratic form $d_k'\mathcal{W}_{yp}d_k$ is negative and group k perceives a negative covariance; if it concentrates on demand-type directions, the covariance is positive.

Corollary 2 (Heterogeneous views, dynamic model)**.** *Households hold a supply-side view and firms a demand-side view whenever their effective loadings lie on opposite sides of the comovement form,*

$$d_h'\,\mathcal{W}_{yp}\,d_h \;<\; 0 \;<\; d_f'\,\mathcal{W}_{yp}\,d_f; \tag{2.27}$$

that is, when the household signal is informative mainly about supply-type directions and the firm signal mainly about demand-type directions.

The simple model is nested as the special case $A = 0$ with $X_t = (q_t, a_t)'$. The effective loading is then $d_k = (b_q\sigma_q^2,\, b_a\sigma_a^2)'$, and with no propagation $\mathcal{W}_{yp} = \Psi_{yp} = \frac{1}{2}(\Psi_y\Psi_p' + \Psi_p\Psi_y')$, where $\Psi_y = (\Psi_{y,q}, \Psi_{y,a})'$ and $\Psi_p = (\Psi_{p,q}, -\Psi_{p,a})'$. Since $d_k'\Psi_{yp}d_k = (\Psi_y'd_k)(\Psi_p'd_k) = \Lambda_y^k\Lambda_p^k$ and $S_k = \operatorname{Var}(s^k)$, (2.25) reduces to the product formula $\operatorname{Cov}(\mathbb{E}_t^k[y_t], \mathbb{E}_t^k[p_t]) = \Lambda_y^k\Lambda_p^k/\operatorname{Var}(s^k)$ of Proposition 3. Away from $A = 0$, X_t contains additional predetermined states, so what each group learns is governed by the full vector d_k.

3 Quantitative Model

In this section, I extend the simple model of Section 2 to a dynamic setting. The objective is threefold: (i) to assess whether the mechanism can quantitatively match the survey evidence documented in Figure 1; (ii) to quantify the consequences of asymmetric attention by households and firms for business cycles; and (iii) to conduct policy experiments.

3.1 Extended model

I extend the simple model of Section 2 along three dimensions. First, I relax the hand-to-mouth assumption and allow households to smooth consumption intertemporally by trading nominal bonds. Second, I introduce strategic complementarity in price setting through segmented labor markets, which matters quantitatively for inflation dynamics. Third, I let the central bank set the nominal interest rate according to a Taylor rule, a more realistic description of monetary policy. The extended model specification is summarized in Appendix D.1.

Households' attention problem In the extended model, the household chooses real bond holdings $\tilde{b}_{i,t}$ and consumption $c_{i,t}$ in each period t. This is equivalent to choosing the vector $x_{i,t}$ in Equation (3.2) if the household knows its own past actions (for the derivation, see Appendix D.2). Formally, household i's rational inattention problem is

$$\max_{s_{i,t}\in\mathcal{S}_h^t} \sum_{t=0}^{\infty} \beta^t\, \mathbb{E}\left[\frac{1}{2}\left(x_{i,t}-x_{i,t}^*\right)'\Theta\left(x_{i,t}-x_{i,t}^*\right)-\mu^h\mathcal{I}\left(\left\{x_{i,t-j}^*\right\}_{j=0}^{\infty};s_{i,t}|S_i^{t-1}\right)|s_i^{-1}\right] \tag{3.1}$$

Here S_i^{t-1} denotes the history of signals up to time $t-1$. And

$$x_{i,t}=\begin{pmatrix}\omega_B\left(\tilde{b}_{i,t}-\tilde{b}_{i,t-1}\right)\\ -\omega_B\left(\frac{1}{\beta}\tilde{b}_{i,t-1}-\tilde{b}_{i,t}\right)+\left(\gamma\frac{\omega_W}{\eta}+1\right)c_{i,t}\end{pmatrix},\quad \Theta=-\bar{C}^{1-\gamma}\begin{bmatrix}\left(\gamma-\frac{\gamma^2\omega_W}{\gamma\omega_W+\eta}\right)\frac{1}{\beta} & 0\\ 0 & \frac{\omega_W}{\gamma\omega_W+\eta}\end{bmatrix} \tag{3.2}$$

Moreover, $x_{i,t}^*$ is the optimal choice vector for household i, which is given by

$$x_{i,t}^*=\begin{pmatrix}z_t-(1-\beta)\sum_{s=t}^{\infty}\beta^{s-t}\mathbb{E}_t\left[z_s\right]+\frac{\beta}{\gamma}\left(1+\omega_W\frac{\gamma}{\eta}\right)\sum_{s=t}^{\infty}\beta^{s-t}\mathbb{E}_t\left(i_s-\pi_{s+1}\right)\\ \omega_W\left(\frac{1}{\eta}+1\right)\tilde{w}_t+\left[\frac{1}{\beta}\omega_B\left(i_{t-1}-\pi_t\right)+\omega_D\tilde{d}_t+\omega_T\tilde{\tau}_t\right]\end{pmatrix} \tag{3.3}$$

The lowercase variables denote the log deviations of the corresponding variables, and variables with a tilde indicate that they are real variables. Moreover, $z_t\equiv\omega_W\left(1+1/\eta\right)\tilde{w}_t+\frac{1}{\beta}\omega_B\left(i_{t-1}-\pi_t\right)+\omega_D\tilde{d}_t+\omega_T\tilde{\tau}_t$. The coefficients $(\omega_B,\omega_W,\omega_D,\omega_T)$ denote the steady-state ratios of $\left(\frac{\bar{B}}{\bar{C}\bar{P}},\frac{\bar{W}\bar{L}}{\bar{C}\bar{P}},\frac{\bar{D}}{\bar{C}\bar{P}},\frac{\bar{T}}{\bar{C}\bar{P}}\right)$.

The first element of the choice vector $x_{i,t}$ is the change in bond holdings, and the second element of $x_{i,t}$ is the component of the marginal rate of substitution between consumption and leisure. These two elements are directly chosen by household through their choice of real bond holdings $\tilde{b}_{i,t}$ and $c_{i,t}$. The formulation of the optimal choice vector (3.3) implies that: (i) it is optimal to increase bond holdings when income is high relative to permanent income or when the real return on bond is high; (ii) it is optimal to equate the marginal rate of substitution between consumption and leisure to the real wage.[18] When the household deviates from these optimal choices, the utility loss is determined by the matrix Θ. This matrix is diagonal, because a suboptimal marginal rate of substitution between consumption and leisure does not affect the optimal change in bond holdings, and a suboptimal change in bond holdings does not affect the optimal marginal rate of substitution between consumption and leisure.

[18] In the formulation, I replaced the labor supply using the budget constraint.

Firms' attention problem After a log-quadratic approximation of the expected discounted profit loss, firm j's rational inattention problem is

$$\max_{s_{j,t}\in\mathcal{S}_f^t}\sum_{t=0}^{\infty}\beta^t\,\mathbb{E}\left[-\frac{\theta-1}{2}\left(p_{j,t}-p_{j,t}^*\right)^2-\mu^f\mathcal{I}\left(p_{j,t}^*;s_{j,t}|S_j^{t-1}\right)|s_j^{-1}\right] \tag{3.4}$$

where

$$p_{j,t}^*=w_{j,t}-a_t=p_t+\alpha\left[y_t-\frac{1+\eta}{\eta+\gamma}a_t\right] \tag{3.5}$$

where $\alpha=\frac{(\eta+\gamma)}{(1+\theta\eta)}$ is the pricing complementarity. Equation (3.5) implies it is optimal for firm j to increase its price if its nominal marginal cost increases, and vice versa.

The equilibrium is defined as in Definition 1, with the additional requirement that the bond market clears.

3.2 Computing the equilibrium

I solve a dynamic general equilibrium model in which both agents are rationally inattentive. The equilibrium is characterized by a fixed-point problem. Specifically, given the processes for the optimal actions of households and firms, $(x_{i,t}^*, p_{j,t}^*)$, I can solve their respective attention problems. At the same time, these processes are themselves determined by the decisions of households and firms. In equilibrium, these two processes must be consistent with each other.

I start by guessing the MA representation of the optimal actions $(x_{i,t}^*, p_{j,t}^*)$ as functions of the productivity (ε_t) and monetary policy (u_t) shocks. I then approximate the processes with truncated MA(200) processes.[19] I then solve the problem numerically using the algorithm for dynamic rational inattention problems (DRIPs) developed in Afrouzi and Yang (2021). Next, I solve the implied state-space representations of other variables in the model, based on which I update the guess for the MA representation of the optimal actions $(x_{i,t}^*, p_{j,t}^*)$, until the model converges.

3.3 Calibration

The model is calibrated at a quarterly frequency. Table 1 reports the calibration.

Assigned parameters. Panel B reports the values of assigned parameters. The values are largely following the literature. I assume the inverse of the Frisch elasticity (η) to be 2.5 and the risk aversion coefficient (γ) to be 3.5, which are standard values in business cycle models. I set the elasticity of substitution across firms (θ) to 10. I use the estimated Taylor rule parameters from Afrouzi and Yang (2021), in particular, they estimate the monetary policy rule for period 1983 to 2007. The persistence is around 0.9457.

[19]With a length of 200, I can get arbitrarily close to the true MA(∞) processes. Increasing the length does not significantly change the results.

Table 1: Calibrated Parameters

Parameter	Value	Moment Matched / Source
Panel A. Calibrated parameters		
Information cost of households μ^h	1.5×10^{-3}	Corr. between $\mathbb{E}^h\,\pi$ and π
Information cost of firms μ^f	0.5×10^{-3}	Corr. between $\mathbb{E}^f\,\pi$ and π
Persistence of productivity shocks (ρ_a)	0.94	Cov. matrix of GDP and inflation
S.D of productivity shocks (σ_a)	1.45×10^{-2}	Cov. matrix of GDP and inflation
Panel B. Assigned parameters		
Time discount factor (β)	0.99	Quarterly frequency
Elasticity of substitution across firms (θ)	10	Firms' average markup
Risk aversion coefficient (γ)	3.5	Households' risk aversion level
Inverse of Frisch elasticity (η)	2.5	Aruoba et al. (2017)
Taylor rule: smoothing (ρ)	0.9457	Estimates 1983–2007
Taylor rule: response to inflation (ϕ_π)	2.028	Estimates 1983–2007
Taylor rule: response to output gap (ϕ_x)	$0.673/4$	Estimates 1983–2007
Taylor rule: response to output growth (ϕ_{dy})	3.122	Estimates 1983–2007
S.D of monetary shocks (σ_u)	0.41×10^{-2}	Romer and Romer (2004)

Calibrated parameters and identification. Panel A reports the four internally calibrated parameters: the household and firm attention costs μ^h and μ^f, and the persistence ρ_a and volatility σ_a of the productivity process. Although the four are calibrated jointly, each is disciplined primarily by a distinct moment. The attention costs are identified from how closely each group's inflation expectations track realized inflation: in the survey data, firms' inflation expectations correlate about 0.7 with realized inflation, whereas households' correlate only about 0.3. A lower attention cost leads a group to acquire more precise signals and raises this correlation, so $\mathrm{Corr}(\mathbb{E}^h\,\pi, \pi)$ identifies μ^h and $\mathrm{Corr}(\mathbb{E}^f\,\pi, \pi)$ identifies μ^f. The calibrated attention cost for households is higher than that for firms, which implies households are more inattentive than firms, which is consistent with additional evidence documented by Coibion et al. (2018); Link et al. (2023). The productivity process (ρ_a, σ_a) is disciplined by the variance–covariance matrix of output and inflation.

3.4 Impulse responses and belief dynamics

I next examine how asymmetric attention shapes both aggregate dynamics and agents' beliefs after structural shocks. I compute impulse responses to supply and demand shocks in the calibrated model. For each shock, I report the responses of aggregate output and inflation, together with the posterior expectations of households and firms. The exercise therefore shows how heterogeneous partial information affects both what agents believe and how aggregate output and inflation respond.

Figure 4 reports the results. The top row shows responses to a positive supply shock. At the aggregate level, output rises and inflation falls. Household beliefs move closely with the realized aggregate responses: households expect higher output and lower inflation, consistent with their greater attention to supply-side information. Firms' beliefs display a more demand-like pattern, with expected inflation and expected output moving together. This reflects firms' demand-biased information set. They misattribute the supply shock as if it were a demand shock.

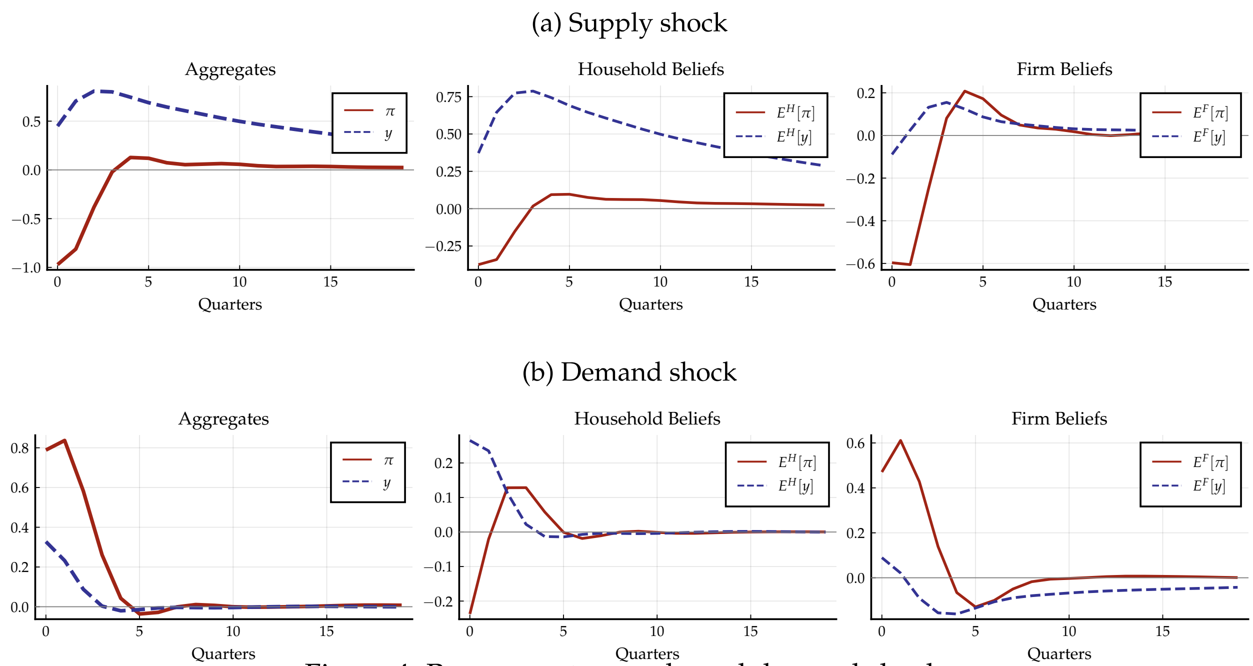


Figure 4: Responses to supply and demand shocks

Notes: The figure plots the IRFs following a one-standard-deviation supply or demand shock.

The bottom row shows responses to an expansionary demand shock. At the aggregate level, output and inflation both rise. Firms' beliefs move more closely with the realized aggregate responses, consistent with their greater attention to demand shocks. Households, however, misread the shock through their supply-biased information set and thus their beliefs temporarily resemble the response to a favorable supply shock.

4 Testing the Mechanism

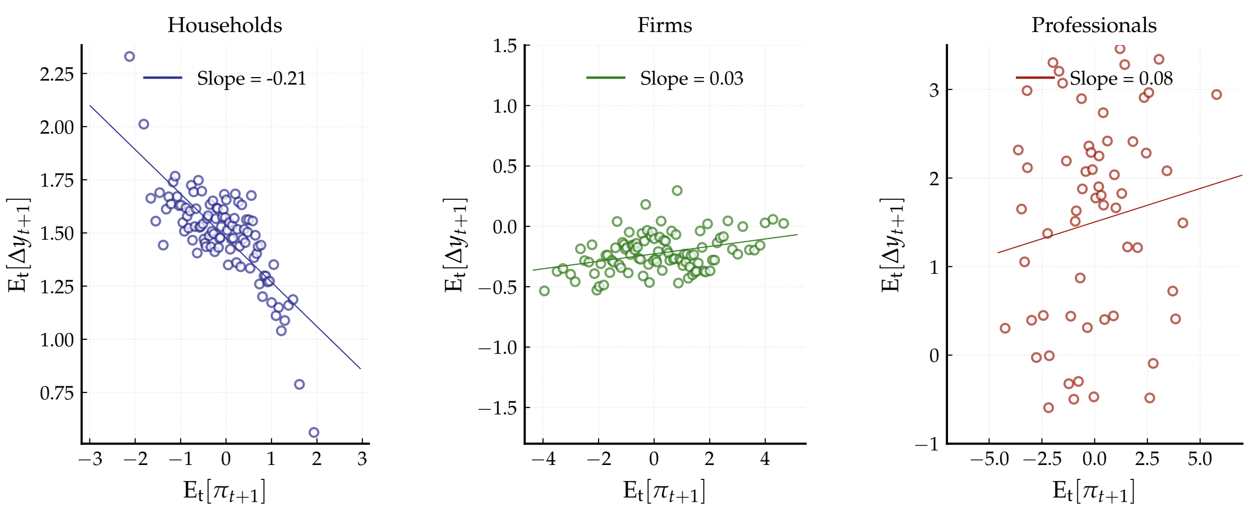
I test the model against two untargeted moments. The first is the unconditional perceived correlation between inflation and real activity for households, firms, and professional forecasters (i.e., Figure 1). The second is dynamic. The model's belief impulse responses imply that households and firms respond asymmetrically to the same aggregate shocks, a prediction that guides an empirical strategy. I estimate the same responses in the survey data, and finding this pattern is, within the logic of the mechanism, "smoking-gun" evidence for it. In addition, as supporting evidence, I show that agents' reported attention choices are important in explaining views, though data limitations confine this to households.

4.1 Perceived output–inflation comovement

The first untargeted moment is agents' perceived comovement between inflation and output, the survey fact in Figure 1: households perceive a negative relationship between expected inflation and expected output growth, whereas firms and professional forecasters perceive a positive one.

To construct the model counterpart, I simulate the calibrated economy $1,000$ times and compute agents' posterior expectations of inflation and output growth. In each simulation, I draw the same number of agents as in the survey data: approximately $1,500$ households per period and 50 firms per period. For each simulated household and firm, I record one-year-ahead expected inflation and expected output growth using their model-implied information sets. I then apply the same empirical specification used to construct the survey figure, plotting expected output growth against expected inflation separately for households, firms, and professional forecasters. Professional forecasters are treated as the full-information benchmark. The resulting model counterpart is shown in Figure 5.

Figure 5: Simulated expected inflation and expected output



Notes: The figure plots the simulated expected inflation and expected output growth for households, firms, and professional forecasters. The parameterization is from Table 1.

Figure 5 presents the model counterpart of Figure 1. The model reproduces the key patterns in the survey data. Households associate higher expected inflation with lower expected output growth, while firms and professional forecasters associate higher expected inflation with higher expected growth. For households, the sign of the slope is the relevant object, since the Michigan Survey of Consumers elicits qualitative rather than quantitative expectations about future business conditions.[20] For firms and professional forecasters, the slope coefficients are quantitatively meaningful. The model matches the negative sign for households and matches both the sign and

[20]In the Michigan Survey of Consumers, respondents are asked whether they expect business conditions over the next year to improve, stay the same, or deteriorate. Following Candia et al. (2020), I assign point values ranging from 1 (improve) to -1 (deteriorate).

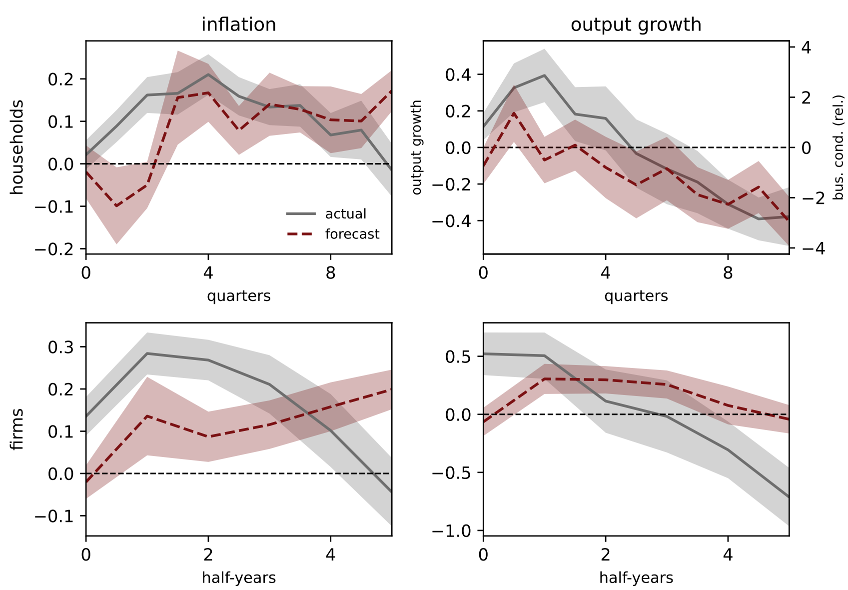
magnitude of the relationships for firms and professional forecasters.

4.2 Belief responses to identified shocks

The model's belief impulse responses imply that households and firms respond asymmetrically to the same aggregate shocks. In particular, the calibrated model implies that households may misread demand shocks as supply shocks, while firms may misread supply shocks as demand shocks. I confront this prediction with the data by estimating, for identified demand and supply shocks, the impulse responses of realized outcomes and of the corresponding one-year-ahead survey forecasts.

To build the empirical counterpart, I identify demand shocks from the high-frequency monetary policy surprises of Nakamura and Steinsson (2018) and supply shocks as technology shocks from a long-run-restricted Galí (1999) VAR. These match the model's two shocks: monetary policy shocks as demand shocks and technology shocks as the supply shocks. For each shock I estimate, by local projection, the response of the realized outcome and of the corresponding one-year-ahead survey forecast, aligning the two to the same target window. The forecasts are from same surveys as Figure 1, and Appendix E gives the details. In each figure the solid line is the realized outcome and the dashed line the corresponding forecast.

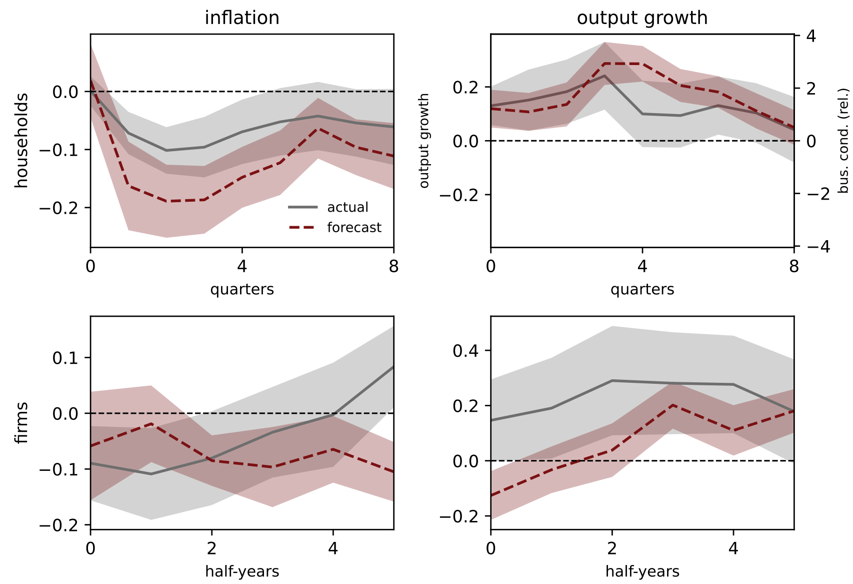
Figure 6: Expectations and outcomes after an expansionary demand (monetary) shock



Notes: Impulse responses to a high-frequency-identified monetary policy (demand) shock, by local projection. Rows are households (quarterly) and firms (semiannual); columns are inflation and output growth. Solid lines are realized outcomes, dashed lines the corresponding one-year-ahead survey forecasts, with 60 percent pointwise confidence bands. The household output-growth panel plots realized GDP growth against the Michigan business-conditions relative score (secondary axis), constructed as the average of point values assigned to each response, from 1 (improve) to −1 (deteriorate).

Figures 6 and 7 report the results. After a demand shock, households' expected inflation falls

Figure 7: Expectations and outcomes after a favorable supply (technology) shock



Notes: Impulse responses to a Galí (1999) technology (supply) shock, by local projection.

on impact even as realized inflation rises, in line with the sign the model predicts, while firms' forecasts follow the direction of the realized responses. After a supply shock, households' forecasts mostly track the realized responses in output and inflation, whereas firms lower their inflation and output expectations, again matching the model's predicted signs.

This exercise should be read only qualitatively, on the sign of the responses. First, the relative business-conditions measure is built from an ordinal survey question,[21] so its level is not comparable to output growth. Moreover, the firm survey is semi-annual against a quarterly model, so the estimates may be noisy. Thus, the data support the belief mechanism in sign, not in magnitude.

4.3 Attention choices and views

As supporting evidence rather than a further test, I show that the attention households report lines up with the view they hold. The mechanism has households attend to the labor market, since the real wage they track moves most with supply shocks. If so, households who pay more attention to labor-market conditions should hold a stronger supply-side view.

The Michigan Survey of Consumers records the news households have heard about the economy, a measure of the developments they have internalized. Households report far more employment-related than price-related news, indicating consistent attention to the real side of the economy. Consistent with the mechanism, households who report attending to employment-related developments hold a more pronounced supply-side view than those who do not. The result survives splitting employment news into favorable and unfavorable items, so it does not simply reflect

[21] Respondents are asked whether they expect business conditions over the next year to improve, stay the same, or deteriorate. The note to Figure 6 describes how the score is constructed.

employment news being more often negative, and it points to attention, not pessimism alone, as a source of the household supply-side view.[22] Appendix A reports the data construction and the regression.

This evidence is available only for households. Direct survey measures linking firm attention to firm views are scarcer, though the Business Inflation Expectations survey indicates that firms attend closely to their costs: between 2011 and 2023, 69 percent of respondents reported that labor costs would affect their prices, consistent with the marginal-cost target the model assigns them.

5 State-Dependent Transmission: The Phillips Curve

An important feature of the model is that households' and firms' attention is endogenous to the economic environment. A shift in monetary policy or in the volatility of shocks changes the value of information on each side, and the resulting reallocation of attention reshapes how shocks pass through to inflation and output.

I apply this mechanism to two episodes in which the slope of the Phillips curve moved in opposite directions. The first is the post-Volcker flattening, which Maćkowiak and Wiederholt (2015) and Afrouzi and Yang (2021) attribute to firm inattention under more hawkish policy. I reproduce it in the two-sided model. The second, and the distinctive contribution of the two-sided model, is the post-pandemic steepening, which requires the interaction between household and firm attention and which firm inattention alone cannot produce.

5.1 Flattening in the post-Volcker period

The post-Volcker period saw a decline in the correlation between inflation and real activity (see Blanchard (2016); Bullard (2018) for example). I show that the shift to more hawkish monetary policy, together with the reallocation of attention by households and firms that it induces, is consistent with the observed flattening of the Phillips curve. This exercise follows the spirit of Maćkowiak and Wiederholt (2015) and Afrouzi and Yang (2021). The main difference here is that the model features rational inattention on both sides.

I use the calibration for the pre-Volcker period from Afrouzi and Yang (2021). The model predicts a flatter Phillips curve under more hawkish policy, along with lower volatility in inflation and output (see Figure 8a).

The intuition is straightforward. First, a more hawkish monetary policy stabilizes inflation, reducing the value of information for firms. Lower firm attention further dampens firms' price adjustment, making inflation less sensitive to output fluctuations and flattening the Phillips curve. Second, firms' lower attention changes the equilibrium variables that enter households' decision problems, such as real wages and real returns. Since households choose signals about these target variables, the information content of their optimal signal depends on how these variables load on

[22] This distinguishes the mechanism from pessimism-based accounts such as Bhandari et al. (2024), in which a common negative bias, rather than what households attend to, drives the negative association between inflation and output.

supply and demand shocks. As firms become less attentive, households' target variables load less on supply shocks and more on demand shocks. Consequently, households' optimal signal also becomes relatively more informative about demand-driven fluctuations. This change in households' information makes demand shocks more important for real activity and contributes to a more volatile output gap. These two attention channels are absent in standard New Keynesian models with full information or exogenous dispersed information. Together, they contribute to the flattening of the Phillips curve.

(a) Post-Volcker flattening (b) Post-COVID steepening

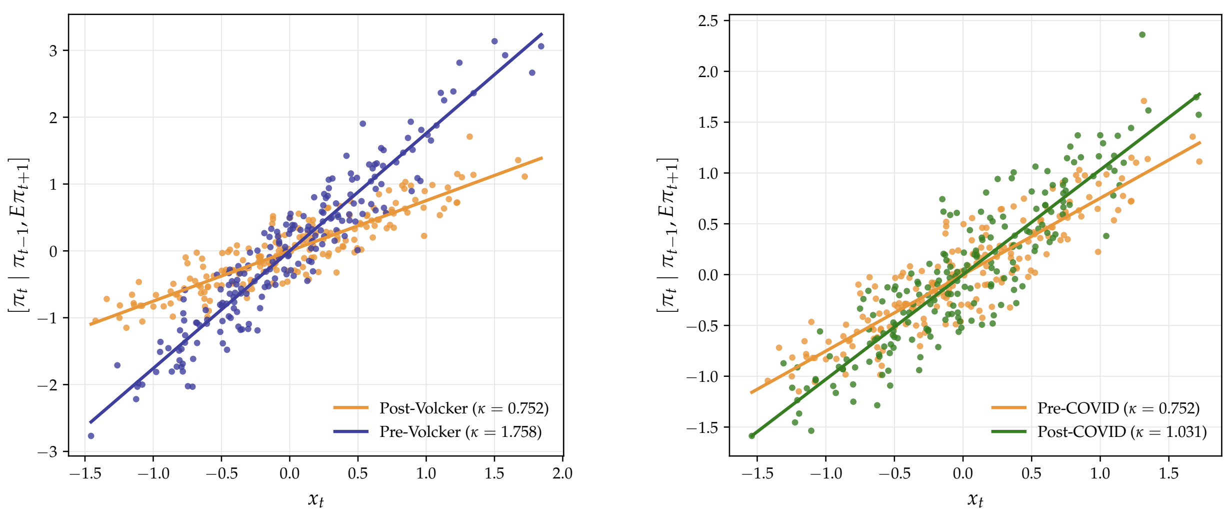


Figure 8: The changing slope of the model-implied Phillips curve

Notes: Panel (a) compares the model-implied Phillips curve under pre- and post-Volcker monetary policies, with the monetary policy parameters for both periods from Afrouzi and Yang (2021). Panel (b) compares the pre-COVID baseline with a post-COVID calibration in which the volatility of both shocks increases by 10 percent. All other parameters are as in Table 1. Each economy is simulated for 1,000 periods.

5.2 Steepening in the post-pandemic period

The post-COVID data signal that the Phillips curve is back (Hobijn et al., 2023; Furlanetto and Lepetit, 2024; Gelain and Lopez, 2024). Using MSA-level data, Cerrato and Gitti (2022) estimate that the slope of the Phillips curve has tripled between the pre-COVID and the post-COVID periods.

As the post-pandemic period was characterized by large supply and demand shocks (Di Giovanni et al., 2023; Shapiro, 2024), I assume a 10 percent increase in the volatility of both shocks in the post-COVID calibration. All other parameters follow the baseline (Table 1). I then simulate the model and compare it with the pre-COVID baseline. Figure 8b shows the results: the model generates a steeper Phillips curve than in the pre-COVID baseline. The exercise is stylized rather than calibrated to the post-COVID data, but it captures the qualitative shift in the slope.

The steepening relies on endogenous attention on both sides of the economy, not on firm inattention alone. To isolate the household channel, I consider a counterfactual in which firms' attention adjusts to the higher-volatility environment while households' signal is held fixed at its baseline. The post-COVID slope is then 0.409, even flatter than the pre-COVID slope of 0.752: with households fixed, the larger shocks raise output-gap volatility and flatten the curve. Only when households also reallocate attention toward supply shocks does output-gap volatility fall and the curve steepen. Changes in household information are therefore crucial for the steepening.

The intuition is that higher shock volatility increases firms' incentive to acquire information, so prices respond more strongly to aggregate conditions. This changes the equilibrium variables that enter households' decision problems, such as real wages and real returns. As firms become more attentive, these household target variables load more strongly on supply shocks and less strongly on demand shocks. Since households choose signals about these target variables, their optimal signal also loads more on supply shocks and less on demand shocks. This change in the information content of households' signal dampens the response of the output gap and is crucial for generating a steeper Phillips curve.

Understanding the dynamics of the slope of the Phillips curve is crucial, as it determines the trade-off between inflation and real activity faced by the monetary authorities. In particular, the steepening of the Phillips curve during the post-COVID period might suggest that contractionary monetary policy can reduce inflation with smaller output losses, and therefore encouraging a stronger policy response to inflation. However, this argument is only partially correct. If households' and firms' attention were fixed, a stronger monetary policy response to inflation could indeed reduce inflation at a limited cost to output. But as discussed in Section 5.1, monetary policy itself can influence agents' attention, and a more hawkish stance flattens the Phillips curve, ultimately worsening the inflation-output trade-off.

6 Communication Policy

I next turn to the implications of heterogeneous attention for communication policy. In particular, I model an announcement as a signal that agents interpret through their own information structure, so that moving the targeted expectation also reshapes their beliefs about the rest of the economy. This is similar to the policy experiment in Kamdar and Ray (2025). However, the point here is that heterogeneous attention choices lead households and firms to interpret the same message differently and, more importantly, that these misaligned interpretations interact to affect aggregate outcomes.

I consider two experiments. In the first, the central bank raises households' and firms' expected future inflation. In the second, it lowers their expected future interest rate. In each case the announcement enters through agents' own signals rather than as a separate observable shock, and I trace how the targeted belief change moves expectations of inflation and output, actions, and the aggregate response.

Formally, let $k \in \{h, f\}$ denote households and firms. In the baseline rational-inattention problem, agent i in group k observes a signal

$$s_{i,t}^k = a_t^{k,*} + e_{i,t}^k, \tag{6.1}$$

where $a_t^{k,*}$ is the full-information optimal action of group k. The announcement is modeled as an additive signal wedge,

$$s_{i,t}^k = a_t^{k,*} + z_{k,t} + e_{i,t}^k. \tag{6.2}$$

Agents do not observe $z_{k,t}$ separately but read it as ordinary news within their own signal. I consider a one-time communication: $z_{k,0} \neq 0$ and $z_{k,t} = 0$ for $t \geq 1$.

6.1 Communication of higher future inflation

For the inflation experiment, I choose $z_{k,0}$ for each group so that the impact response of expected future inflation is the same for households and firms, $\Delta\mathbb{E}_t^h[\pi_{t+1}] = \Delta\mathbb{E}_t^f[\pi_{t+1}] = \Delta\bar{\pi}$.

Household beliefs determine households' output decision, which gives the direct output response, and firm beliefs determine firms' reset-price decision, which gives the implied inflation response through price aggregation. I also compute a one-round firm-to-household feedback that captures how the firm pricing response changes households' target. The resulting output response can be written as

$$y_t^{total} = y_t^{direct} + y_t^{indirect}, \tag{6.3}$$

where y_t^{direct} is the household response induced directly by the manipulated household posterior, and $y_t^{indirect}$ is the additional response induced by firms' price setting through the household target. This is not a full general-equilibrium fixed point but a direct household-belief channel plus a one-round firm-price feedback. Because both groups are inattentive, each further feedback round is attenuated by their signal responses, both well below one, so the one-round object captures the bulk of the general-equilibrium response.

Figure 9 reports the response to an announcement that raises expected future inflation by $\Delta\bar{\pi} = 0.05$ on impact. Households and firms both revise perceived inflation upward on impact.[23] The output-belief responses reveal the main mechanism. Households read the higher inflation as a contractionary supply shock, so their expected output falls sharply on impact. Firms instead read it as higher demand, so their expected output rises. The same announcement thus generates heterogeneous revisions in the perceived joint distribution of inflation and output.

[23] Perceived inflation then gradually reverses toward zero. This reversal reflects the Kalman-implied adjustment of posterior beliefs after the one-time signal manipulation. The dynamics differ because households and firms have different information structures.

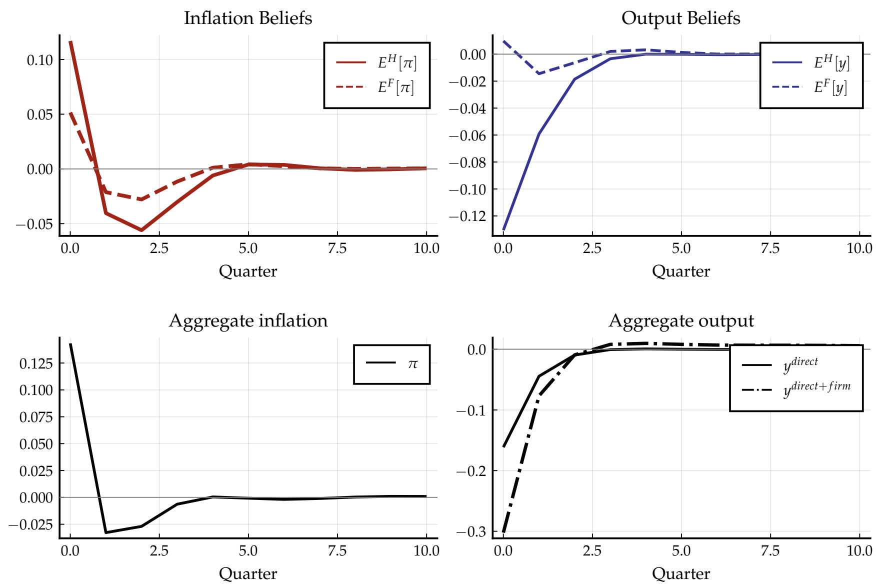


Figure 9: Communication of higher future inflation

Notes: The figure plots the impulse responses of households' and firms' expected inflation and expected output, as well as the aggregate inflation and output, following the communication of a higher future inflation.

The aggregate responses inherit these belief differences. Firms raise reset prices on impact, generating a positive inflation response. Households reduce their output action, producing a negative direct output response. The one-round firm feedback further amplifies the initial output decline, so that y_t^{total} lies below y_t^{direct} at short horizons. The contraction is short-lived and fades within a few quarters.

The aggregate output response contrasts sharply with the standard full-information benchmark. Under the usual Euler-equation logic, higher expected inflation lowers the perceived real interest rate, holding nominal rates fixed, and should stimulate current consumption. In the model, however, households do not interpret the announcement as a pure change in expected inflation. They process it through their own information structure and infer bad real news. As a result, the direct household response is contractionary. This is even worse as firms raise reset prices in response to the same announcement, and this firm-price response further lowers the aggregate output.

6.2 Communication of a lower interest rate

I next consider communication about the policy rate. The exercise studies how households and firms interpret the announcement's information content, rather than the mechanical effect of an actual rate change. I normalize the announcement so that it lowers both groups' expected future interest rate on impact by the same amount, $\Delta\mathbb{E}_t^h[i_{t+1}] = \Delta\mathbb{E}_t^f[i_{t+1}] = \Delta\bar{i}$ with $\Delta\bar{i} = -0.05$. The communication is one-time, and beliefs then evolve according to agents' filtering equations. To isolate its informational content from the standard real-interest-rate channel, I hold the policy-rate response in the aggregate block fixed, so that no actual rate change accompanies the announcement.

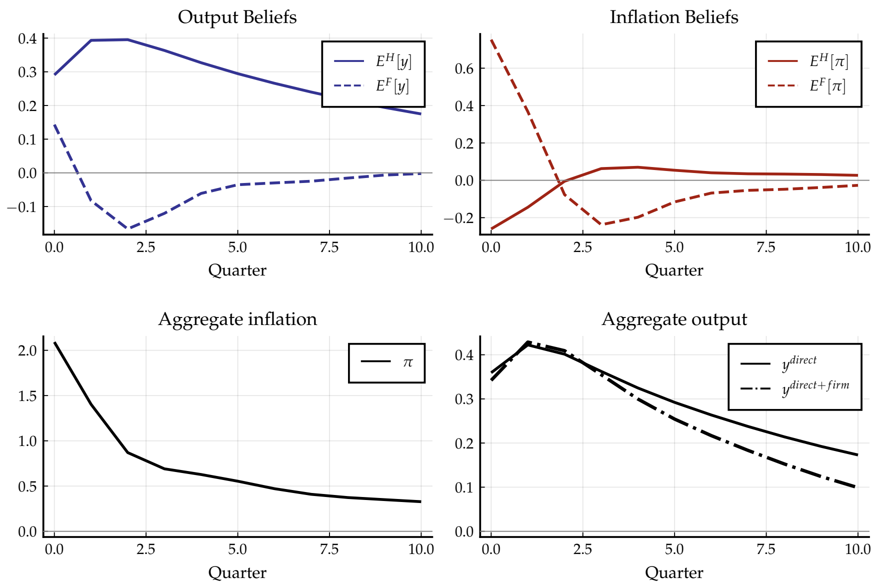


Figure 10: Communication of a lower interest rate

Notes: The figure plots the impulse responses of households' and firms' expected inflation and expected output, as well as the aggregate inflation and output, following the communication of a lower interest rate.

Figure 10 reports the results. As in the inflation experiment, the same announcement produces sharply different belief revisions. Households read the lower rate as the central bank's endogenous response to an expansionary supply shock, so their expected output rises and their expected inflation falls. Firms instead read it as a response to stronger demand, so both their expected output and their expected inflation rise. Output expectations move up for both groups, while inflation expectations diverge, falling for households and rising for firms.

The mapping from beliefs to outcomes parallels the inflation experiment. Households' posterior determines their consumption response, and firms' posterior determines their reset-price decision and hence aggregate inflation. As households revise output up they consume more and raise aggregate output, and as firms revise inflation up they raise prices. The one-round firm-price feedback amplifies output slightly at short horizons but dampens it at longer ones: in the figure, y_t^{total} tracks y_t^{direct} on impact and falls below it later. The firm-price channel therefore does not overturn the expansionary direct household response, but it shortens its persistence.

7 Conclusion

This paper develops a model of two-sided rational inattention in which households and firms choose what to pay attention to given their distinct decision problems. Households care about the real wage and firms about their nominal marginal cost, so the two groups optimally attend to different combinations of supply and demand shocks. The resulting attention is heterogeneous, endogenous, and strategic, as each group's information choice reshapes the signals the other faces. This mechanism accounts for the central puzzle in the survey data, that households read high inflation as a sign of weak real activity while firms read it as a sign of strong demand.

Because attention responds to the policy regime and to the volatility of shocks, the transmission of shocks is state-dependent. The same reallocation of attention flattens the Phillips curve under the more hawkish post-Volcker policy and steepens it in the high-volatility post-pandemic period, with the household side essential for the steepening. Heterogeneous attention also reshapes communication policy. Because households and firms interpret the same announcement through different information structures, an announcement that moves a targeted expectation can move their beliefs about output and inflation in opposite directions, and these conflicting revisions can leave the economy worse off than intended.

The framework points to a broader agenda. The forces that make attention differ across households and firms also operate within each group, so agents that differ in income, education, or size should differ systematically in what they attend to and in the views they hold. Mapping this within-group heterogeneity to micro data, and tracing its implications for the distributional effects of monetary policy, is a natural next step.

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A Supplementary Evidence on Expectations and Attention

This appendix documents the robustness of the cross-sectional views (Section 1) and the attention regression (Section 4.3).

A.1 Robustness of the cross-sectional views

The cross-sectional pattern of Figure 1 also holds within respondents. Using the panel dimension of the surveys, I run the forecast regression separately for each respondent observed at least three times. Of the $4{,}276$ households interviewed at least three times in the Michigan Survey of Consumers, 75.3 percent display a negative slope, so that when a household raises its inflation forecast between interviews it also forecasts weaker activity. For firms and professional forecasters the slope is more often positive: 54.3 percent of firms with at least five observations, and 73.7 percent of professional forecasters with at least ten.

A.2 Attention choices and views

The Michigan Survey of Consumers asks respondents what economic changes they have heard about recently, which captures the information they have internalized.[24] Households report employment-related news far more consistently than price-related news, indicating persistent attention to the real side of the economy.

To see whether attention shapes views, I estimate

$$\begin{aligned}\mathbb{E}_t^i[Growth] = \beta_0 + \beta_1\,\mathbb{E}_t^i[Inflation] + \gamma_1\,\mathbb{E}_t^i[Inflation] \times News_{i,t}^{labor} + \gamma_2\,\mathbb{E}_t^i[Inflation] \times News_{i,t}^{price} \\ + \alpha_1 News_{i,t}^{labor} + \alpha_2 News_{i,t}^{price} + \alpha_t + u_{i,t},\end{aligned} \tag{A1}$$

where $News_{i,t}^{labor}$ and $News_{i,t}^{price}$ indicate whether respondent i reports hearing labor-market or price news. A supply-side view corresponds to $\beta_1 < 0$, and $\gamma_1 < 0$ means that attention to labor news strengthens it. Table A.1 reports $\beta_1 < 0$ and $\gamma_1 < 0$, both significant: households who attend to labor-market news hold a stronger supply-side view. Attention to price news works weakly in the opposite direction ($\gamma_2 > 0$, insignificant). The labor-news result is unchanged when I split labor news into favorable and unfavorable items (columns 2 and 3), so it does not reflect labor news being disproportionately negative, and it rules out pessimism as the sole source of the household supply-side view.

On the firm side, comparable panel measures linking attention to views are unavailable, but the Business Inflation Expectations survey indicates that firms attend to their costs: between 2011 and 2023, 69 percent of respondents reported that labor costs would affect their prices, consistent with the marginal-cost target the model assigns them.

[24] A description of the question and the list of categories is available from the Michigan Survey of Consumers. I sort responses into employment-related and price-related news, each split into favorable and unfavorable items.

Table A.1: Perceived relationship between inflation and growth: households

	Growth forecasts		
	All	labor news (+)	labor news (−)
Inflation forecast	-0.047^{***}	-0.047^{***}	-0.047^{***}
	(0.001)	(0.001)	(0.001)
Inflation forecast × labor news	-0.019^{**}	-0.019^{*}	-0.013^{*}
	(0.007)	(0.010)	(0.008)
Inflation forecast × price news	0.006	0.006	0.006
	(0.027)	(0.027)	(0.027)
labor news	-0.091^{***}	0.150^{***}	-0.237^{***}
	(0.025)	(0.025)	(0.022)
price news	0.061	0.061	0.061
	(0.073)	(0.073)	(0.073)
Intercept	0.019	0.019	0.020
	(0.002)	(0.002)	(0.002)
Observations	218,716	217,936	218,295
R^2	0.022	0.022	0.022

Note: Estimates of regression (A1). Column 1 uses the full sample; columns 2 and 3 split labor news into favorable and unfavorable items. Time fixed effects are included and standard errors are in parentheses. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

B Proofs for the Simple Model

B.1 Second-order approximation of objectives

Both objectives are approximated by the same envelope argument, which I state once and then apply to each agent.

Lemma 1 (Quadratic loss). Let an agent choose a scalar x to maximize a smooth objective $\hat{f}(x, z)$, where z collects the variables it takes as given, and let $x^*(z) \equiv \arg\max_x \hat{f}(x, z)$ satisfy the interior first-order condition $\hat{f}_x(x^*, z) = 0$. Then, to second order around the non-stochastic steady state,

$$\hat{f}(x, z) = \tfrac{1}{2}\,\hat{f}_{xx}\,(x - x^*)^2 + \big(\text{terms independent of } x\big) + \mathcal{O}\big(\|x, z\|^3\big)\,, \tag{A1}$$

where $\hat{f}_{xx}$ is evaluated at the steady state.

Proof. Define $L(x, z) \equiv \hat{f}(x, z) - \hat{f}(x^*, z)$. A second-order Taylor expansion of L in x about x^* has a vanishing first-order term because $\hat{f}_x(x^*, z) = 0$, so $L(x, z) = \frac{1}{2}\hat{f}_{xx}(x - x^*)^2 + \mathcal{O}(\|\cdot\|^3)$. Adding back $\hat{f}(x^*, z)$, which does not depend on x, gives (A1). □

It remains, for each agent, to (i) reduce the objective to a function of its own action and the variables it takes as given, (ii) evaluate the steady-state curvature $\hat{f}_{xx}$, and (iii) record the optimal action x^*.

Households. Because households are hand-to-mouth, labor supply follows from the budget constraint, $L_{i,t} = P_t C_{i,t}/W_t$, so period utility is a function of the choice $c_{i,t}$ and the taken-as-given

$\{w_t, p_t\}$ alone:

$$\hat{u}\left(c_{i,t}, p_t, w_t\right) = \frac{\left(\bar{C}e^{c_{i,t}}\right)^{1-\gamma}}{1-\gamma} - \frac{1}{1+\eta}\left(\frac{\bar{P}e^{p_t}\bar{C}e^{c_{i,t}}}{\bar{W}e^{w_t}}\right)^{1+\eta}.$$

At the steady state $\hat{u}_{cc} = -(\gamma+\eta)$, and the first-order condition $\hat{u}_c = 0$ gives $c^*_{i,t} = \lambda\left(w_t - p_t\right)$ with $\lambda = (1+\eta)/(\gamma+\eta)$. Lemma 1 then yields

$$\hat{u}\left(c_{i,t}, p_t, w_t\right) = -\frac{\gamma+\eta}{2}\left(c_{i,t} - c^*_{i,t}\right)^2 + \left(\text{terms independent of } c_{i,t}\right) + \mathcal{O}(\|\cdot\|^3).$$

Firms. Substituting the production function $Y_{j,t} = A_t L_{j,t}$ and the demand curve $Y_{j,t} = (P_{j,t}/P_t)^{-\theta} Y_t$ into the profit (2.3), firm j's profit is a function of the choice $p_{j,t}$ and the taken-as-given $\{p_t, w_t, y_t, a_t\}$; up to a factor that does not depend on $p_{j,t}$ it is proportional to

$$\hat{\pi}\left(p_{j,t}; p_t, w_t, y_t, a_t\right) \;\propto\; e^{-\theta(p_{j,t}-p_t)}\left[e^{p_{j,t}} - \left(1-\theta^{-1}\right)e^{w_t - a_t}\right].$$

At the steady state $\hat{\pi}_{pp} = -(\theta-1)$, and the first-order condition $\hat{\pi}_p = 0$ gives $p^*_{j,t} = w_t - a_t$. Lemma 1 then yields

$$\hat{\pi}\left(p_{j,t}; \cdot\right) = -\frac{\theta-1}{2}\left(p_{j,t} - p^*_{j,t}\right)^2 + \left(\text{terms independent of } p_{j,t}\right) + \mathcal{O}(\|\cdot\|^3).$$

These are the loss functions used in the attention problems (2.6) and (2.9).

B.2 Proof of Proposition 1

Both agents' attention problems, (2.6) and (2.9), are instances of the following linear-quadratic-Gaussian result.

Lemma 2 (LQG attention). An agent chooses a signal s_t about a Gaussian target T_t (mean zero, prior variance σ^2_T) to solve

$$\max_{s_t}\; \mathbb{E}\Big[\tfrac{1}{2}\hat{f}_{xx}\left(x_t - x^*_t\right)^2 - \mu\,\mathcal{I}(T_t; s_t)\Big], \qquad x^*_t = \phi\, T_t,\;\; \hat{f}_{xx} < 0,\;\; \mu > 0,$$

where x_t is the action and x^*_t its full-information optimum. Let $\kappa \equiv -\hat{f}_{xx}\phi^2 > 0$. The optimal policy is a single Gaussian signal about the target,

$$s_t = T_t + e_t, \qquad e_t \sim \mathcal{N}\Big(0,\; \tfrac{1-\xi}{\xi}\sigma^2_T\Big), \quad e_t \perp T_t,$$

with attention level $\xi = \max\{0,\; 1 - \mu/(\kappa\sigma^2_T)\} \in [0,1]$, and action $x_t = \phi\,\xi\,(T_t + e_t)$.

Proof. For any signal, the action that maximizes the expected payoff is the posterior mean, $x_t = \mathbb{E}[x^*_t \mid s_t] = \phi\,\mathbb{E}[T_t \mid s_t]$, so the expected payoff loss is $-\frac{1}{2}\hat{f}_{xx}\phi^2\,\mathbb{E}[\mathrm{Var}(T_t \mid s_t)] = \frac{1}{2}\kappa\,\sigma^2_{T|s}$, where $\sigma^2_{T|s}$ is the posterior variance. Following Maćkowiak et al. (2018), the optimal signal can be restricted to a single signal about T_t. For a Gaussian prior and quadratic loss, the signal and posterior are Gaussian, so $\mathcal{I}(T_t; s_t) = \frac{1}{2}\log(\sigma^2_T/\sigma^2_{T|s})$. The problem can thus be reformulated as the standard LQG problem:

$$\max_{\sigma^2_{T|s} \le \sigma^2_T}\; -\frac{1}{2}\kappa\,\sigma^2_{T|s} - \frac{1}{2}\mu\log\left(\frac{\sigma^2_T}{\sigma^2_{T|s}}\right),$$

For an interior solution, the first-order condition is $-\frac{\kappa}{2}+\frac{\mu}{2\sigma^2_{T|s}}=0$, solving gives $\sigma^2_{T|s}=\mu/\kappa$. Accounting for the constraint $\sigma^2_{T|s}\le\sigma^2_T$, the optimal posterior variance is $\sigma^2_{T|s}=\min\{\sigma^2_T,\ \mu/\kappa\}$.

Define the agent's attention level as the Kalman gain. It follows from the relationship between prior and posterior variance, $\xi=1-\sigma^2_{T|s}/\sigma^2_T=\max\{0,\ 1-\mu/(\kappa\sigma^2_T)\}$, which is zero when $\mu\ge\kappa\sigma^2_T$ and approaches one as $\mu\to 0$. For $\xi>0$ this posterior is delivered by the signal $s_t=T_t+e_t$ with the stated noise variance, and Gaussian updating gives $\mathbb{E}[T_t\mid s_t]=\xi(T_t+e_t)$, hence $x_t=\phi\,\xi(T_t+e_t)$. □

Proof of Proposition 1. By Lemma 1, each agent's payoff loss is quadratic in the gap between its action and full-information action, so both attention problems are instances of Lemma 2, under the mapping

$$\text{Households:}\quad (x_t,\hat f_{xx},\phi,T_t,\mu)=\big(c_{i,t},\ -(\gamma+\eta),\ \lambda,\ w_t-p_t,\ \mu^h\big),\qquad \sigma^2_T=\sigma^2_\omega,$$

$$\text{Firms:}\quad (x_t,\hat f_{xx},\phi,T_t,\mu)=\big(p_{j,t},\ -(\theta-1),\ 1,\ w_t-a_t,\ \mu^f\big),\qquad \sigma^2_T=\sigma^2_{mc}.$$

Hence $\kappa_h=-\hat f_{xx}\phi^2=(\gamma+\eta)\lambda^2$ for households and $\kappa_f=\theta-1$ for firms. Substituting into Lemma 2 delivers the optimal signals, the attention levels ξ_h and ξ_f in (2.10), and the induced actions stated in Proposition 1. □

B.3 Proof of Proposition 2

Aggregating the individual actions of Proposition 1 (the idiosyncratic signal noise averages out) gives the aggregate rules (2.11), $c_t=\lambda\xi_h(w_t-p_t)$ and $p_t=\xi_f(w_t-a_t)$. Goods-market clearing is $y_t=c_t$, and the definition of nominal aggregate demand $Q_t=P_tY_t$ gives, in logs, $q_t=p_t+y_t=p_t+c_t$. Substituting the aggregate rules,

$$q_t=\xi_f(w_t-a_t)+\lambda\xi_h(w_t-p_t).$$

Using $p_t=\xi_f(w_t-a_t)$ to write $w_t-p_t=(1-\xi_f)w_t+\xi_f a_t$ and collecting terms in w_t,

$$q_t=D\,w_t-\xi_f(1-\lambda\xi_h)\,a_t,\qquad D\equiv\xi_f+\lambda\xi_h(1-\xi_f),$$

so that $w_t=\big[q_t+\xi_f(1-\lambda\xi_h)a_t\big]/D$. Substituting w_t back gives the two equilibrium targets,

$$w_t-p_t=\frac{1-\xi_f}{D}\,q_t+\frac{\xi_f}{D}\,a_t,\qquad w_t-a_t=\frac{1}{D}\,q_t-\frac{\lambda\xi_h}{D}\,a_t,$$

which are (2.12). By Proposition 1 each agent observes its target plus idiosyncratic noise, which yields the signals in parts 1 and 2.

Because q_t and a_t are independent, the equilibrium target volatilities are

$$\sigma^2_\omega=\frac{(1-\xi_f)^2\sigma^2_q+\xi_f^2\sigma^2_a}{D^2},\qquad \sigma^2_{mc}=\frac{\sigma^2_q+\lambda^2\xi_h^2\sigma^2_a}{D^2}.$$

Substituting these into the interior attention levels of Proposition 1, $1-\xi_h=\mu^h/\big[(\gamma+\eta)\lambda^2\sigma^2_\omega\big]$ and $1-\xi_f=\mu^f/\big[(\theta-1)\sigma^2_{mc}\big]$, yields the fixed point (2.15).

Existence and uniqueness. Write the equilibrium as the fixed point $\xi = \Phi(\xi)$ of the best-response operator $\Phi : [0,1]^2 \to [0,1]^2$,

$$\Phi_k(\xi) = \max\left\{0,\ 1 - \frac{\mu^k}{\kappa_k\, \sigma_k^2(\xi)}\right\}, \qquad k \in \{h, f\},$$

with $\kappa_h = (\gamma + \eta)\lambda^2$, $\kappa_f = \theta - 1$, and $\sigma_h^2 \equiv \sigma_\omega^2$, $\sigma_f^2 \equiv \sigma_{mc}^2$ the target volatilities above. Since each $\sigma_k^2(\xi)$ is continuous and positive on $\{D > 0\}$, Φ is a continuous self-map of the compact convex set $[0,1]^2$, so a fixed point exists by Brouwer's theorem. On the interior ($\Phi_k > 0$), differentiating and using $1 - \Phi_k(\xi) = \mu^k/(\kappa_k \sigma_k^2)$ gives

$$\frac{\partial \Phi_k}{\partial \xi_j} = \big(1 - \Phi_k(\xi)\big)\, \frac{\partial \ln \sigma_k^2(\xi)}{\partial \xi_j},$$

so, since $1 - \Phi_k \le 1$, the Jacobian satisfies $\|D\Phi(\xi)\|_\infty \le \max_k \sum_j \left|\partial \ln \sigma_k^2/\partial \xi_j\right|$. The log-sensitivities are bounded away from the no-attention corner. Hence, whenever

$$\max_{k\in\{h,f\}} \sum_{j\in\{h,f\}} \left|\frac{\partial \ln \sigma_k^2(\xi)}{\partial \xi_j}\right| < 1 \qquad \text{on the interior region,}$$

Φ is a contraction on $([0,1]^2, \|\cdot\|_\infty)$ and the interior equilibrium is its *unique* fixed point. This condition can fail only as $D \to 0$, where the target volatilities become arbitrarily sensitive to attention; any multiplicity is therefore confined to corner (zero-attention) configurations. □

B.4 Proof of Proposition 4

Throughout I work on the interior region in which both attention levels are strictly positive, so that the best-response operator of Appendix B.3 satisfies $\xi_k = \Phi_k(\xi) = 1 - \mu^k/(\kappa_k \sigma_k^2)$ with $\kappa_h = (\gamma + \eta)\lambda^2$, $\kappa_f = \theta - 1$, and $\sigma_h^2 \equiv \sigma_\omega^2$, $\sigma_f^2 \equiv \sigma_{mc}^2$. Differentiating,

$$\frac{\partial \xi_h}{\partial \xi_f} = \frac{\mu^h}{\kappa_h(\sigma_\omega^2)^2}\, \frac{\partial \sigma_\omega^2}{\partial \xi_f}, \qquad \frac{\partial \xi_f}{\partial \xi_h} = \frac{\mu^f}{\kappa_f(\sigma_{mc}^2)^2}\, \frac{\partial \sigma_{mc}^2}{\partial \xi_h}, \tag{A2}$$

so each best-response slope inherits the sign of the derivative of the *other* group's target volatility, and it suffices to sign these derivatives shock by shock. Recall from (2.12) the equilibrium loadings

$$b_{h,q} = \frac{1 - \xi_f}{D}, \quad b_{h,a} = \frac{\xi_f}{D}, \quad b_{f,q} = \frac{1}{D}, \quad b_{f,a} = -\frac{\lambda \xi_h}{D}, \qquad D \equiv \xi_f + \lambda \xi_h (1 - \xi_f),$$

with $\lambda \in (0,1)$ when $\gamma > 1$, and $\partial D/\partial \xi_f = 1 - \lambda \xi_h > 0$, $\partial D/\partial \xi_h = \lambda(1 - \xi_f) > 0$.

Demand shocks only ($\sigma_a^2 = 0$). The household target reduces to $w_t - p_t = b_{h,q} q_t$, so $\sigma_\omega^2 = b_{h,q}^2 \sigma_q^2$ and $\text{sign}(\partial \sigma_\omega^2/\partial \xi_f) = \text{sign}\big(b_{h,q}\, \partial b_{h,q}/\partial \xi_f\big)$. Since $b_{h,q} > 0$ and

$$\frac{\partial b_{h,q}}{\partial \xi_f} = \frac{-D - (1 - \xi_f)(1 - \lambda \xi_h)}{D^2} < 0,$$

the target volatility falls in ξ_f, so by (A2) $\partial \xi_h/\partial \xi_f \le 0$: the attention levels are strategic substitutes. Symmetrically the firm target is $w_t - a_t = b_{f,q} q_t$ with $\sigma_{mc}^2 = b_{f,q}^2 \sigma_q^2$, and $b_{f,q} = 1/D$ is decreasing in ξ_h because $\partial D/\partial \xi_h > 0$; hence $\partial \xi_f/\partial \xi_h \le 0$, the same sign.

Supply shocks only ($\sigma_q^2 = 0$). Now $w_t - p_t = b_{h,a} a_t$, so $\sigma_\omega^2 = b_{h,a}^2 \sigma_a^2$ with $b_{h,a} = \xi_f / D > 0$ and

$$\frac{\partial b_{h,a}}{\partial \xi_f} = \frac{D - \xi_f \, \partial D / \partial \xi_f}{D^2} = \frac{\lambda \xi_h}{D^2} \geq 0,$$

strictly positive whenever $\xi_h > 0$. Thus σ_ω^2 rises in ξ_f and $\partial \xi_h / \partial \xi_f > 0$: the attention levels are strategic complements. For the firm, $w_t - a_t = b_{f,a} a_t$ with $|b_{f,a}| = \lambda \xi_h / D$ and

$$\frac{\partial |b_{f,a}|}{\partial \xi_h} = \lambda \, \frac{D - \xi_h \, \partial D / \partial \xi_h}{D^2} = \frac{\lambda \xi_f}{D^2} \geq 0,$$

so σ_{mc}^2 rises in ξ_h and $\partial \xi_f / \partial \xi_h > 0$, again the same sign. $\square$

B.5 Proof of Proposition 5

Level effect. With both shocks present the household target volatility is $\sigma_\omega^2 = b_{h,q}^2 \sigma_q^2 + b_{h,a}^2 \sigma_a^2 = \big[(1 - \xi_f)^2 \sigma_q^2 + \xi_f^2 \sigma_a^2\big] / D^2$, so

$$\frac{\partial \sigma_\omega^2}{\partial \xi_f} = \underbrace{2 \, b_{h,q} \, \frac{\partial b_{h,q}}{\partial \xi_f} \, \sigma_q^2}_{\leq 0} + \underbrace{2 \, b_{h,a} \, \frac{\partial b_{h,a}}{\partial \xi_f} \, \sigma_a^2}_{\geq 0}.$$

By the single-shock computations the demand term is nonpositive and the supply term nonnegative, so their sum—and hence $\partial \xi_h / \partial \xi_f$ through (A2)—takes the sign of whichever term dominates: it is negative when σ_q^2 is large relative to σ_a^2 and positive in the opposite case, and is ambiguous in general. The same decomposition applied to $\sigma_{mc}^2 = \big[\sigma_q^2 + \lambda^2 \xi_h^2 \sigma_a^2\big] / D^2$ shows that $\partial \xi_f / \partial \xi_h$ is likewise ambiguous, establishing the level claim.

Composition effect. The relative informativeness of a signal about the two shocks is measured by the ratio of its supply to its demand loading, in which the common factor $1/D$ cancels. For the household,

$$\frac{b_{h,a}}{b_{h,q}} = \frac{\xi_f}{1 - \xi_f}, \qquad \frac{\partial}{\partial \xi_f} \frac{b_{h,a}}{b_{h,q}} = \frac{1}{(1 - \xi_f)^2} > 0,$$

so the ratio is strictly increasing in firm attention: a rise in ξ_f tilts the household signal toward supply. For the firm,

$$\left| \frac{b_{f,a}}{b_{f,q}} \right| = \lambda \xi_h, \qquad \frac{\partial}{\partial \xi_h} \big(\lambda \xi_h \big) = \lambda > 0,$$

so a rise in household attention tilts the firm signal toward supply. Hence, although the effect of one group's attention on the other's attention level is ambiguous in sign, its effect on the composition of the affected signal is not: greater attention by either group shifts the informational content of the signals toward supply relative to demand. $\square$

B.6 Proof of Proposition 3

The shocks q_t, a_t are mean-zero and independent with variances σ_q^2, σ_a^2, and the signal noise $e_{k,t}$ is mean-zero and independent of (q_t, a_t). All variables are jointly Gaussian, so agent k's conditional expectation of any variable is its linear projection on the signal $s_{k,t}$; for output,

$$\mathbb{E}^k[y_t] = \frac{\mathrm{Cov}(y_t, s_{k,t})}{\mathrm{Var}(s_{k,t})}\, s_{k,t},$$

and analogously for p_t. Because both expectations are proportional to the same signal $s_{k,t}$,

$$\mathrm{Cov}\big(\mathbb{E}^k[y_t], \mathbb{E}^k[p_t]\big) = \frac{\mathrm{Cov}(y_t, s_{k,t})\,\mathrm{Cov}(p_t, s_{k,t})}{\mathrm{Var}(s_{k,t})}.$$

Using $y_t = \Psi_{y,q}q_t + \Psi_{y,a}a_t$, $p_t = \Psi_{p,q}q_t - \Psi_{p,a}a_t$, and $s_{k,t} = b_{k,q}q_t + b_{k,a}a_t + e_{k,t}$ together with the independence of $q_t, a_t, e_{k,t}$,

$$\begin{aligned}\mathrm{Cov}(y_t, s_{k,t}) &= \Psi_{y,q}b_{k,q}\sigma_q^2 + \Psi_{y,a}b_{k,a}\sigma_a^2 = \Lambda_y^k\\ \mathrm{Cov}(p_t, s_{k,t}) &= \Psi_{p,q}b_{k,q}\sigma_q^2 - \Psi_{p,a}b_{k,a}\sigma_a^2 = \Lambda_p^k\end{aligned}$$

and $\mathrm{Var}(s_{k,t}) = b_{k,q}^2\sigma_q^2 + b_{k,a}^2\sigma_a^2 + \mathrm{Var}(e_{k,t}) > 0$. Substituting gives (2.18). Since $\mathrm{Var}(s_{k,t}) > 0$, the sign of the covariance equals that of $\Lambda_y^k\Lambda_p^k$, which is positive when the two projections share a sign and negative when they differ. □

C Dynamic Characterization of Joint Beliefs

This appendix proves Proposition 6 and Corollary 2, shows that the simple model of Section 2 is the special case $A = 0$, extends the characterization to vector actions, and records the joint state-space system used to compute all actual and perceived second moments. Throughout, the transition matrices (A, Q), the response vectors (Ψ_y, Ψ_p), the loadings B_k, the prior covariances $\Sigma_{1|0}^k$, and the signal noise variances R^k are equilibrium objects: they are determined jointly with the attention choices of households and firms at the fixed point, not taken as primitives.

C.1 Set-up and filtering

The state evolves as in (2.22), $X_t = AX_{t-1} + Q\varepsilon_t$ with $\varepsilon_t \sim \mathcal{N}(0, I)$, and A is stable (all eigenvalues inside the unit circle) in a stationary equilibrium. An agent in group $k \in \{h, f\}$ tracks the target $a_t^{k,*} = B_kX_t$ using the single Gaussian signal $s_{i,t}^k = B_kX_t + e_{i,t}^k$ with $e_{i,t}^k \sim \mathcal{N}(0, R^k)$. Write the one-step-ahead prior covariance as $\Sigma_{1|0}^k \equiv \mathrm{Var}(X_t \mid \mathcal{I}_{t-1}^k)$, and define

$$d_k \equiv \Sigma_{1|0}^k B_k', \qquad S_k \equiv B_k\Sigma_{1|0}^k B_k' + R^k > 0. \tag{A1}$$

In a stationary equilibrium $\Sigma_{1|0}^k$ is the stabilizing solution of the algebraic Riccati equation $\Sigma_{1|0}^k = A\big(\Sigma_{1|0}^k - d_kd_k'/S_k\big)A' + QQ'$, in which d_k and S_k from (A1) themselves depend on $\Sigma_{1|0}^k$. The Kalman filter updates the posterior mean as $\hat{X}_{i,t}^k = A\hat{X}_{i,t-1}^k + K_k\,\nu_{i,t}^k$, with Kalman gain and innovation

$$K_k = \Sigma_{1|0}^k B_k'/S_k = d_k/S_k, \qquad \nu_{i,t}^k = s_{i,t}^k - B_kA\hat{X}_{i,t-1}^k. \tag{A2}$$

The innovation is scalar, serially uncorrelated, and has variance $\mathrm{Var}(\nu_{i,t}^k) = S_k$. The filtered state is therefore a first-order autoregression driven by a single scalar innovation,

$$\hat{X}_{i,t}^k = A\hat{X}_{i,t-1}^k + \frac{d_k}{S_k}\,\nu_{i,t}^k. \tag{A3}$$

The perceived moments below are individual-level: they are the second moments of an agent's own beliefs and therefore include the agent's idiosyncratic signal noise through $K_k\nu_{i,t}^k$, consistent with the individual survey responses the model is mapped to. Under a continuum of agents with independent noise, the corresponding group-average moments are obtained by dropping the idiosyncratic-noise term. I suppress the individual index i where it is immaterial.

C.2 Proof of Proposition 6

Perceived output and prices are linear in the filtered state, $\mathbb{E}_t^k[y_t] = \Psi_y'\hat{X}_t^k$ and $\mathbb{E}_t^k[p_t] = \Psi_p'\hat{X}_t^k$, so

$$\mathrm{Cov}\Big(\mathbb{E}_t^k[y_t], \mathbb{E}_t^k[p_t]\Big) = \Psi_y' M_k \Psi_p, \qquad M_k \equiv \mathrm{Var}(\hat{X}_t^k). \tag{A4}$$

The innovation in (A3) is orthogonal to the date-$(t-1)$ information set, and hence to $\hat{X}_{t-1}^k$. Taking the variance of (A3), the cross term vanishes, and imposing stationarity $\mathrm{Var}(\hat{X}_t^k) = \mathrm{Var}(\hat{X}_{t-1}^k) = M_k$ gives the Lyapunov equation

$$M_k = AM_kA' + J_k, \qquad J_k \equiv \frac{d_k}{S_k}\,\mathrm{Var}(\nu_t^k)\,\frac{d_k'}{S_k} = \frac{d_kd_k'}{S_k}, \tag{A5}$$

where J_k is the covariance of the one-period belief revision $\hat{X}_t^k - A\hat{X}_{t-1}^k$. Since A is stable, (A5) has the unique solution

$$M_k = \sum_{j\geq 0} A^j J_k (A')^j. \tag{A6}$$

Because (A4) is a scalar it equals its symmetric part, $\Psi_y' M_k \Psi_p = \mathrm{Tr}(M_k\Psi_{yp})$ with $\Psi_{yp} \equiv \frac{1}{2}(\Psi_y\Psi_p' + \Psi_p\Psi_y')$. Substituting (A6) and using the cyclic property of the trace to move $(A')^j$ to the front of each term,

$$\mathrm{Tr}\big(M_k\Psi_{yp}\big) = \sum_{j\geq 0}\mathrm{Tr}\big(\Psi_{yp}A^jJ_k(A')^j\big) = \sum_{j\geq 0}\mathrm{Tr}\big((A')^j\Psi_{yp}A^j\,J_k\big) = \mathrm{Tr}\big(\mathcal{W}_{yp}J_k\big), \tag{A7}$$

where

$$\mathcal{W}_{yp} \equiv \sum_{j\geq 0}(A')^j\Psi_{yp}A^j \tag{A8}$$

is the unique solution of the dual Lyapunov equation $\mathcal{W}_{yp} = \Psi_{yp} + A'\mathcal{W}_{yp}A$. Finally, since $J_k = d_kd_k'/S_k$,

$$\mathrm{Tr}\big(\mathcal{W}_{yp}J_k\big) = \frac{\mathrm{Tr}\big(\mathcal{W}_{yp}d_kd_k'\big)}{S_k} = \frac{d_k'\mathcal{W}_{yp}d_k}{S_k}.$$

Combining (A4)–(A7) gives

$$\mathrm{Cov}\Big(\mathbb{E}_t^k[y_t], \mathbb{E}_t^k[p_t]\Big) = \frac{d_k'\,\mathcal{W}_{yp}\,d_k}{S_k},$$

which is (2.25). Since $S_k > 0$, the sign of the perceived covariance is the sign of the quadratic form $d_k' \mathcal{W}_{yp} d_k$, establishing (2.26). $\square$

C.3 The comovement matrix and Corollary 2

The kernel $\mathcal{W}_{yp}$ in (A8) accumulates the contemporaneous output–price comovement Ψ_{yp} over the entire propagation of the state. The matrix is symmetric but generally indefinite. Its eigenvectors with positive eigenvalues span the "demand-like" directions, along which a disturbance moves output and prices together, and its eigenvectors with negative eigenvalues span the "supply-like" directions, along which they move in opposite directions. The quadratic form $d_k' \mathcal{W}_{yp} d_k$ then takes the sign of whichever set of directions dominates the projection of d_k: it is positive when d_k loads mainly on the demand-like directions and negative when it loads mainly on the supply-like ones. Corollary 2 is the statement that the two groups fall on opposite sides, $d_h' \mathcal{W}_{yp} d_h < 0 < d_f' \mathcal{W}_{yp} d_f$, which by (2.26) makes the household's perceived covariance negative and the firm's positive.

C.4 The simple model as the case $A = 0$

Set $A = 0$ and $X_t = (q_t, a_t)' = Q\varepsilon_t$, so that $\Sigma^k_{1|0} = QQ' = \mathrm{diag}(\sigma_q^2, \sigma_a^2)$. With loadings $B_k = (b_{k,q}, b_{k,a})$ the effective loading is $d_k = \Sigma^k_{1|0} B_k' = (b_{k,q}\sigma_q^2,\ b_{k,a}\sigma_a^2)'$ and $S_k = b_{k,q}^2\sigma_q^2 + b_{k,a}^2\sigma_a^2 + R^k = \mathrm{Var}(s^k)$. Because there is no propagation, the dual Lyapunov equation collapses to $\mathcal{W}_{yp} = \Psi_{yp} = \frac{1}{2}(\Psi_y\Psi_p' + \Psi_p\Psi_y')$, with $\Psi_y = (\Psi_{y,q},\ \Psi_{y,a})'$ and $\Psi_p = (\Psi_{p,q}, -\Psi_{p,a})'$. The quadratic form then factors,

$$d_k' \Psi_{yp} d_k = (\Psi_y' d_k)(\Psi_p' d_k) = \Lambda^k_y \Lambda^k_p,$$

so (2.25) reduces to $\mathrm{Cov}(\mathbb{E}^k_t[y_t], \mathbb{E}^k_t[p_t]) = \Lambda^k_y \Lambda^k_p / \mathrm{Var}(s^k)$, the product formula of Proposition 3.

C.5 Vector actions

Suppose group k chooses q_k actions, so that B_k is $q_k \times n$, the signal is the vector $s^k_{i,t} = B_k X_t + e^k_{i,t}$ with $e^k_{i,t} \sim \mathcal{N}(0, R^k)$ for a $q_k \times q_k$ covariance R^k, and $S_k = B_k \Sigma^k_{1|0} B_k' + R^k$ is now a matrix. The Kalman gain is $K_k = d_k S_k^{-1}$ with $d_k = \Sigma^k_{1|0} B_k'$, and the filtered-state Lyapunov equation (A5) holds with $J_k = K_k S_k K_k' = d_k S_k^{-1} d_k'$. Repeating the trace algebra of Appendix C.2 gives

$$\mathrm{Cov}\Big(\mathbb{E}^k_t[y_t], \mathbb{E}^k_t[p_t]\Big) = \mathrm{Tr}\big(\mathcal{W}_{yp}\, d_k S_k^{-1} d_k'\big) = \mathrm{Tr}\big(d_k' \mathcal{W}_{yp} d_k\, S_k^{-1}\big). \tag{A9}$$

The $q_k \times q_k$ matrix $d_k' \mathcal{W}_{yp} d_k$ is symmetric but generally indefinite, while $S_k^{-1} \succ 0$. Unlike the scalar case, a positive-definite S_k^{-1} does not fix the sign of (A9): by Sylvester's law of inertia congruence preserves the inertia of $d_k' \mathcal{W}_{yp} d_k$, but $\mathrm{Tr}(MP)$ for $P \succ 0$ is sign-definite only when M is semidefinite. The signal precision S_k^{-1} reweights how much each action contributes, so when a group's signals span both comovement cones the sign of its perceived covariance depends on that precision structure. The scalar-action case in the main text is the leading case in which $d_k' \mathcal{W}_{yp} d_k$ is a scalar and the sign is unambiguous.

C.6 Joint system and cross-agent moments

The proposition characterizes each group's within-group perceived covariance in closed form. To compute actual moments and cross-agent objects – for instance the disagreement measures used in Section 6 – stack the state and the two filtered states into $Z_t \equiv (X_t', \hat{X}_t^{h\prime}, \hat{X}_t^{f\prime})'$. Combining (2.22)–(A3) for each group, and writing the innovation as $\nu_t^k = B_k Q \varepsilon_t + e_t^k + B_k A(X_{t-1} - \hat{X}_{t-1}^k)$, the joint system is a first-order Gaussian law of motion

$$Z_t = \mathcal{A} Z_{t-1} + \mathcal{C} w_t, \qquad w_t \equiv (\varepsilon_t', e_t^h, e_t^f)', \quad \Omega \equiv \mathrm{Var}(w_t), \tag{A10}$$

where $\mathcal{A}$ and $\mathcal{C}$ are assembled from A, Q, and the gains $K_h = d_h/S_h$, $K_f = d_f/S_f$; for example the household block reads $\hat{X}_t^h = K_h B_h A X_{t-1} + (I - K_h B_h) A \hat{X}_{t-1}^h + K_h B_h Q \varepsilon_t + K_h e_t^h$. The stationary covariance Σ_Z solves the Lyapunov equation $\Sigma_Z = \mathcal{A}\Sigma_Z\mathcal{A}' + \mathcal{C}\Omega\mathcal{C}'$, and every actual and perceived second moment is a linear functional of Σ_Z. The within-group covariance $\Psi_y' M_k \Psi_p$ is read from the diagonal block $M_k = \mathrm{Var}(\hat{X}_t^k)$, recovering (2.25); a cross-agent covariance such as $\mathrm{Cov}(\mathbb{E}_t^h[y_t], \mathbb{E}_t^f[p_t]) = \Psi_y' \mathrm{Cov}(\hat{X}_t^h, \hat{X}_t^f)\Psi_p$ is read from the corresponding off-diagonal block.

D Extended Model: Derivations

D.1 Extended model

Households. There is a continuum of households, indexed by $i \in [0,1]$. Each period, household i chooses the consumption level $C_{i,t}$ and bond holdings $B_{i,t}$ based on its information set $\mathcal{S}_i^t = \{s_{i,\tau}\}_{\tau=0}^t$. After deciding on consumption and bond holdings, household i supplies labor $L_{i,t}$ at the given wage W_t so that the budget constraint holds. Formally, household i's expected present value of utility is

$$\mathbb{E}^i\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \frac{L_{i,t}^{1+\eta}}{1+\eta}\right)\right], \quad C_{i,t} = \left[\int_0^1 C_{i,j,t}^{\frac{\theta-1}{\theta}}\,dj\right]^{\frac{\theta}{\theta-1}} \tag{A1}$$

less the cost of attention. The budget constraint is

$$P_t C_{i,t} + B_{i,t} = W_t L_{i,t} + R_{t-1} B_{i,t-1} + D_t + T_t \tag{A2}$$

Here B_t is the nominal bond holdings at t that yield a nominal return of R_t at $t+1$, D_t is the aggregate profits of firms, and T_t is the net lump-sum transfers (or taxes, if negative). Household i takes $\{P_t, R_t, W_t, D_t, T_t\}$ as given.

Firms. There is a continuum of firms producing differentiated goods, indexed by $j \in [0,1]$. Firm j faces the demand curve $Y_{j,t} = (P_{j,t}/P_t)^{-\theta} Y_t$. Firm j takes the wage $W_{j,t}$ and the demand for its good as given. In each period, firm j sets the price of its variety $P_{j,t}$ based on its information and then hires labor $L_{j,t}$ to meet demand through the production function $Y_{j,t} = A_t L_{j,t}$. Formally, firm j's expected present value of profit, discounted by households' marginal utility of consumption, is

$$\mathbb{E}^j\left[\sum_{t=0}^{\infty} C_t^{-\gamma}\left[P_{j,t}Y_{j,t} - (1-\theta^{-1})\frac{W_{j,t}}{A_t}\left(\frac{P_{j,t}}{P_t}\right)^{-\theta} Y_t\right]\right] \tag{A3}$$

less the cost of attention. Here A_t is aggregate productivity, with $a_t \equiv \log A_t$ following an AR(1) process $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t$, $\varepsilon_t \sim N(0,1)$. Other variables are defined as in Section 2.

Central bank. The central bank has full information: it knows the shocks, households' and firms' actions, and the equilibrium outcomes. Monetary policy is a standard Taylor rule with interest-rate smoothing,

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho} \left[\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\phi_x} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_{dy}}\right]^{1-\rho} e^{-\sigma_u u_t} \tag{A4}$$

where R_t is the nominal interest rate, $\bar{R}$ the steady-state nominal rate, $Y_t = C_t$ aggregate output, Y_t^n the natural level of output, and $u_t \sim N(0,1)$ a monetary policy shock, specified so that a positive u_t is expansionary. Denoting $i_t \equiv \log R_t$, the log-linearised rule is

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_x x_t + \phi_{dy} \Delta y_t\right) - u_t. \tag{A5}$$

I interpret the central bank as the counterpart of the professional forecasters in the survey.

Fiscal authority. The government finances maturing nominal bonds and the wage subsidy by collecting lump-sum taxes or issuing new bonds, subject to

$$\frac{B_t}{P_t} = \frac{R_{t-1}}{\Pi_t}\frac{B_{t-1}}{P_{t-1}} + \theta^{-1}\frac{W_t L_t}{P_t} + \frac{T_t}{P_t}.$$

I consider two assumptions about how the government satisfies its intertemporal budget constraint: (i) government debt is held constant and transfers fully adjust; (ii) government debt absorbs the majority of the fiscal imbalance in the short run, and the path of lump-sum taxes adjusts to satisfy long-run solvency. Following common practice in the New Keynesian literature, I restrict the tax rule so that monetary policy is active and fiscal policy is passive in the sense of Leeper (1991).

D.2 Approximation of households' utility

Using the flow budget constraint (A2) to substitute for labor in the utility function and expressing all variables as log-deviations from the non-stochastic steady state, the per-period utility of household i in period t is

$$u = \frac{\bar{C}^{1-\gamma}}{1-\gamma} e^{(1-\gamma)c_{i,t}} - \frac{1}{1+\eta}\left[\frac{\bar{P}\bar{C}e^{p_t + c_{i,t}} + \bar{B}e^{b_{i,t}} - \bar{R}\bar{B}e^{i_{t-1}+b_{i,t-1}} - \bar{D}e^{d_t} - \bar{T}e^{\tau_t}}{\bar{W}e^{w_t}}\right]^{1+\eta}$$

where lowercase letters denote log-deviations: $c_{i,t}$ is consumption, $\tilde{b}_{i,t}$ real bond holdings, $\tilde{d}_t$ real dividends, and $\tilde{\tau}_t$ real transfers. Define the steady-state ratios

$$(\omega_B, \omega_W, \omega_D, \omega_T) = \left(\frac{\bar{B}}{\bar{C}\bar{P}}, \frac{\bar{W}\bar{L}}{\bar{C}\bar{P}}, \frac{\bar{D}}{\bar{C}\bar{P}}, \frac{\bar{T}}{\bar{C}\bar{P}}\right).$$

In period t household i chooses $v_t \equiv (\tilde{b}_{i,t}, c_{i,t})'$; the previous period's choices are $v_{t-1} = (\tilde{b}_{i,t-1}, 0)'$. Households take $\zeta_t \equiv [\tilde{d}_t, i_{t-1}, \tilde{w}_t, \tilde{\tau}_t, \pi_t]'$ as given.

A log-quadratic approximation to the expected discounted sum of per-period utility around the non-stochastic steady state yields

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}_i^h \left[\frac{1}{2}\left(v_t - v_t^*\right)' \Theta_0 \left(v_t - v_t^*\right) + \left(v_t - v_t^*\right)' \Theta_1 \left(v_{t+1} - v_{t+1}^*\right)\right] \tag{A6}$$

where

$$\Theta_0 = -\bar{C}^{1-\gamma} \begin{bmatrix} \frac{\eta}{\omega_W}\left[1+\frac{1}{\beta}\right]\omega_B^2 & \frac{\eta}{\omega_W}\omega_B \\ \frac{\eta}{\omega_W}\omega_B & \gamma + \frac{\eta}{\omega_W} \end{bmatrix}, \qquad \Theta_1 = \bar{C}^{1-\gamma} \begin{bmatrix} \frac{\eta}{\omega_W}\omega_B^2 & \frac{\eta}{\omega_W}\omega_B \\ 0 & 0 \end{bmatrix}.$$

The full-information optimal bond holdings satisfy

$$\omega_B\left(\tfrac{1}{\beta}\tilde{b}^*_{i,t-1} - \tilde{b}^*_{i,t}\right) + c^*_{i,t} = \mathbb{E}_t\left[\omega_B\left(\tfrac{1}{\beta}\tilde{b}^*_{i,t} - \tilde{b}^*_{i,t+1}\right) + c^*_{i,t+1}\right] \tag{A7}$$

and the optimal consumption choice

$$-\omega_B\left(\tfrac{1}{\beta}\tilde{b}^*_{i,t-1} - \tilde{b}^*_{i,t}\right) + \left(\gamma\tfrac{\omega_W}{\eta}+1\right)c^*_{i,t} = \omega_W\left(\tfrac{1}{\eta}+1\right)\tilde{w}_t + \left[\tfrac{1}{\beta}\omega_B\left(i_{t-1}-\pi_t\right) + \omega_D\tilde{d}_t + \omega_T\tilde{\tau}_t\right]. \tag{A8}$$

Together with the log-linearised budget constraint

$$c_{i,t} = \omega_W\left(\tilde{w}_t + l_{i,t}\right) + \tfrac{1}{\beta}\omega_B\left(i_{t-1}-\pi_t\right) + \omega_B\left(\tfrac{1}{\beta}\tilde{b}_{i,t-1} - \tilde{b}_{i,t}\right) + \omega_D\tilde{d}_t + \omega_T\tilde{\tau}_t, \tag{A9}$$

combining (A8) with (A7) under full information yields the intertemporal Euler equation $c^*_{i,t} = \mathbb{E}_t[c^*_{i,t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1})]$, and combining (A9) with (A8) gives the intratemporal condition

$$\tilde{w}_t = \gamma c^*_{i,t} + \eta l^*_{i,t}. \tag{A10}$$

To solve for optimal bond holdings under full information I follow the transformation in Maćkowiak and Wiederholt (2025): substituting (A10) into (A9), summing from t to ∞ with discount β^t, taking expectations, and using the Euler equation and the law of iterated expectations gives

$$\omega_B\tilde{b}^*_{i,t} = \omega_B\tilde{b}^*_{i,t-1} + z_t - (1-\beta)\sum_{s=t}^{\infty}\beta^{s-t}\mathbb{E}_t\left[z_s\right] + \left(1+\omega_W\tfrac{\gamma}{\eta}\right)\tfrac{1}{\gamma}\sum_{s=t+1}^{\infty}\beta^{s-t}\mathbb{E}_t\left(r_{s-1}-\pi_s\right), \tag{A11}$$

where $z_t \equiv \omega_W(1+\frac{1}{\eta})\tilde{w}_t + \frac{1}{\beta}\omega_B(i_{t-1}-\pi_t) + \omega_D\tilde{d}_t + \omega_T\tilde{\tau}_t$.

The off-diagonal element of Θ_0 in (A6) is non-zero, so a suboptimal bond holding $\tilde{b}^*_{i,t}$ affects the optimal consumption choice $c^*_{i,t}$ and vice versa, while the second term of (A6) links today's bond holdings to tomorrow's. To remove these intra- and inter-temporal dependencies, I follow Maćkowiak and Wiederholt (2025) and let household i choose the transformed vector

$$x_{i,t} = \begin{pmatrix} \omega_B\left(\tilde{b}_{i,t} - \tilde{b}_{i,t-1}\right) \\ -\omega_B\left(\frac{1}{\beta}\tilde{b}_{i,t-1} - \tilde{b}_{i,t}\right) + \left(\gamma\frac{\omega_W}{\eta}+1\right)c_{i,t} \end{pmatrix},$$

which diagonalises the loss function,

$$\sum_{t=0}^{\infty}\beta^t\mathbb{E}_i^h\left[\tfrac{1}{2}(v_t - v_t^*)'\Theta_0(v_t - v_t^*) + (v_t - v_t^*)'\Theta_1(v_{t+1} - v_{t+1}^*)\right] = \sum_{t=0}^{\infty}\beta^t\mathbb{E}_{i,-1}\left[\tfrac{1}{2}(x_{i,t} - x^*_{i,t})'\Theta(x_{i,t} - x^*_{i,t})\right], \tag{A12}$$

with Θ and $x^*_{i,t}$ given in Equation (3.2) and (3.3).

E Impulse Responses of Outcomes and Expectations

Section 4.2 compares the dynamic responses of realized outcomes and of the corresponding survey forecasts to identified structural shocks. For each shock and each object, the realized outcome or the one-year-ahead forecast, I estimate the impulse response horizon by horizon and plot the two together, so that the gap between them measures how far expectations depart from the realized path. The design follows Angeletos et al. (2021).

Data. Household expectations are from the Michigan Survey of Consumers: the mean expected rate of price change over the next twelve months, and the expected change in business conditions summarized by the standard diffusion (relative) score, aggregated to quarterly frequency. Firm expectations are the mean one-year-ahead CPI and real-output-growth forecasts from the Livingston Survey. Realized inflation is the twelve-month CPI change and realized output growth the four-quarter change in real GDP, each dated so that the observation at t is the outcome realized over the year the t-dated forecast predicts.

Shocks. The demand shock is the high-frequency-identified monetary policy surprise of Nakamura and Steinsson (2018), aggregated from event days to the quarter. The supply shock is a technology shock identified as in Galí (1999): I estimate a bivariate structural VAR in the growth rates of labor productivity and hours worked, taken from Fernald (2014), and impose the long-run restriction that only technology shocks move the level of labor productivity permanently, the Blanchard and Quah (1989) identification applied to Galí's variables.

Specification. For each standardized shock ε_t, object z, and horizon h, I estimate the local projection (Jordà, 2005)

$$z_{t+h} = \alpha_h + \beta_h \, \varepsilon_t + \gamma_h' \, X_t + u_{t+h},$$

and report the sequence $\{\beta_h\}$ as the impulse response. The forecast and the realized object refer to the same one-year window at every h. Because a survey is fielded before the period's shock is fully realized, I shift the forecast projections by one survey round relative to the realized projections. Controls X_t are lags of the outcome and of the forecast, standard errors are heteroskedasticity-robust, and the shaded bands are 60 percent pointwise confidence intervals. The household output-growth panels plot realized GDP growth against the business-conditions relative score, an ordinal index on a secondary axis, and are read for comovement and timing rather than for magnitude.